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The Limits of Price  
Information in  
Market Processes

by

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THE LIMITS OF PRICE INFORMATION IN MARKET PROCESSES.

by

Avraham Beja

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# THE LIMITS OF PRICE INFORMATION IN MARKET PROCESSES.

## I. INTRODUCTION.

In economic systems, outcomes depend upon the actions of agents and are influenced by the agents' beliefs about the environment. Inferences from observed data help agents form their assessments about not-immediately-observable (but relevant) characteristics, such as the prospects of stocks in future markets, the durability of a given product, etc. One way to formalize this is by considering a whole class of possible environments. Typically, agents have only partial information as to which environment they are in. Their decisions will generally depend on the information they have, and, consequently, so will prices. Suppose the system associates an equilibrium price with each environment. Traders who are familiar with the process can augment this in their information structure and make inferences from prices about the information available to other traders. This raises a number of interesting questions:

1) Would the price signals generate allocations that are efficient with respect to the traders' preferences, conditional on the aggregate information available to all traders?

2) Would the existence of equilibria in "classical" environments still be assured?

3) How would the possibility of making inferences from prices affect traders' willingness to engage in observation of the environment and in the acquisition of information?

All the above issues are obviously interrelated. Radner (1957) has

shown that if the information revealed by prices is as good as the joint information possessed by all traders the answer to (1) above is positive and the price system is indeed efficient. On the other hand, examples have been used to show that when traders make conclusive inferences from prices equilibrium may fail to exist even in relatively simple economies (e.g. Green 1975, Kreps 1975, Jordan 1976). When equilibrium prices do exist, the inferential opportunities they afford have an impact on the profitability of the search for new information. Superior information has distributional implications, and it has been suggested (cf. Hirschleifer 1971, 1973) that this may lead to a socially wasteful "over investment" in information. If, however, information is well reflected in easily observable prices, traders' behavior will resemble a market with homogenous information, where speculative trading originating from diverse beliefs can hardly take place (cf. Grossman and Stiglitz 1975). Redistributive opportunities are then severely limited, and with them much of the incentive for traders to acquire costly information. The "production efficiency" of the system is then impaired, because with little acquisition of information prices will at best provide only limited signals for efficient production decisions.

This paper is concerned with a basic issue in price information, which is crucial to all the above questions, i.e. what is the extent of the inferences that can be derived from prices or, equivalently, how well can a given piece of information be inferred from prices? This question has attracted considerable direct interest in financial economics, because of the obvious immediate implications for investment policies in the stockmarket (e.g. Fama 1970, and Black 1971) or for the firms' reporting

policies (e.g. Ball and Brown 1968). The literature is especially concerned with situations in which information is "fully reflected" in prices. Radner's condition represents an extreme case, in which all information can be inferred from prices with complete accuracy. Weaker inferences apply when the price is a "sufficient statistic" for some (but not necessarily "all") information. Even though the data cannot be fully and accurately deduced from the observed prices, the potential inferences can still be considered operationally conclusive. This aspect was studied by Kihlstrom and Mirman (1975), who presented conditions for the price to be a sufficient statistic for some information. Grossman (1976) and Grossman and Stiglitz (1975) have constructed neat models that exhibit this property. In these models, a price function on the environment is shown to satisfy the equilibrium properties and to be a sufficient statistic for all available information.

Limiting the analysis to the equilibrium properties of a hypothetical price function may overlook an important question. Consideration must be given also to the existence of some process whereby such a function could be derived from the traders' behavior in the marketplace. Systems in which such a process exists, so that prices depend on the environment only through traders' demands, are termed here "genuine trading processes". The results of this paper show that in genuine trading processes prices cannot be meaningfully "fully informative". They cannot accurately reflect any useful information, nor even be sufficient statistics for it. In equilibrium, the only information which can be accurately inferred from prices is inferior information which can also be accurately inferred from other data; prices can (trivially) be "sufficient statistics" only for

basically useless information which would not affect traders' decisions even in non-informative systems.

## II. THE MODEL.

Consider an economy with  $I$  traders,<sup>1</sup> indexed by  $i=1, \dots, I$ , trading in  $J$  assets whose price is denoted by  $p=(p^1, p^2, \dots, p^J)$ . Each trader determines his position  $x=(x^1, x^2, \dots, x^J)$  according to his possibilities, his tastes, and his beliefs about the environment. The relevant aspects of environmental uncertainty are represented by the set  $W$  of states of the world  $w$ , an unknown one of which obtains. Diverse beliefs about  $W$  may induce traders with similar tastes and possibilities to make different decisions. Information and inferences that change initial beliefs are central to our analysis. Traders with diverse backgrounds may draw different inferences from the same data. Following Marschak (1959, 1963), this is incorporated in the model by an extended description  $s=(w, z)$  of the states of nature, where  $w$  incorporates the aspects that are relevant for (final) decisions and  $z$  the aspects that are relevant for inferences.<sup>2</sup> Traders have their (heterogeneous) beliefs about the set  $S$  of possible (extended descriptions  $s$  of the) states of nature, which clearly also involve their beliefs about  $W$ . Information about the environment can be described as a function on  $S$ . When a trader knows that  $F(s)=f$ , he

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<sup>1</sup>To lend the system competitive properties, it suffices to think of  $I$  classes of traders, rather than  $I$  traders, cf. Grossman (1976).

<sup>2</sup>Marschak calls  $w$  the "external" or "payoff relevant" state and  $z$  the state of the "information instrument".

restricts his attention to those states in  $S$  that could have generated  $f$  under the function  $F$ , and correspondingly updates his probability assessments for the relevant aspects  $w$  of the states  $s$ . Diverse beliefs about  $S$  allow traders to make diverse (probabilistic) inferences about  $w$  from the same information.<sup>3</sup> For any set  $X$  of tentative market positions (a subset of  $R^J$ ) and any collection  $F_1, F_2, \dots$  of functions on  $S$ , let (the  $J$ -dimensional vector)  $G_1[X|F_1(s)=f_1, F_2(s)=f_2, \dots]$  denote the  $i^{\text{th}}$  trader's preferred position among the alternatives in  $X$ , if he knows that  $F_1(s)=f_1, F_2(s)=f_2$  etc.<sup>4</sup> The formulation explicitly separates the two considerations that are involved in the trader's choice - the range of feasible alternatives, and the information that guides his choice.

Traders' information is heterogeneous, and they do not all have access to the same data. Generally, there are  $N$  types of observations, where  $y_n = Y_n(s)$  means that when the state is  $s$  the data  $y_n$  is generated by the observation  $Y_n$ . For convenience, we adopt the following notation convention. Let  $\underline{N} = \{1, 2, \dots, N\}$  denote the set of all observations, and for any subset  $A$  of  $\underline{N}$  (written  $A \subseteq \underline{N}$ ), let  $Y_A = \{Y_n, n \in A\}$  denote the set of

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<sup>3</sup>To see how one information function on  $S$  can mean different things to different traders, consider this simple example. Suppose (the value of)  $w$  can be either  $G$  (good) or  $B$  (bad), and suppose an economic indicator can be either  $H$  (high) or  $L$  (low). Suppose further that two traders initially consider  $G$  and  $B$  equally likely, but whereas one makes no inferences from the economic indicator the other associates  $G$  with  $H$  and  $B$  with  $L$ . This can be described with a model where  $S = \{GH, GL, BH, BL\}$  and  $F(w, z) = z$  regardless of  $w$ , where the first trader initially assigns probability 0.25 to all states  $s$ , and the other trader assigns probability 0.5 to  $GH$  and  $BL$  and zero probability to  $GL$  and  $BH$ .

<sup>4</sup>With utility maximizing and price taking traders, this is the position  $x^*$  that maximizes the conditional expected utility, given  $F_1(s)=f_1, F_2(s)=f_2$  etc., over all  $x$  in  $X$ . If  $x^*$  is not unique,  $G_1(X| \dots)$  may be defined either as the set of all utility maximizing solutions, or just one solution selected by the  $i^{\text{th}}$  trader through any arbitrary rule.



observations with indexes in  $A$  and  $y_A = \{y_n, n \in A\}$  the data generated by these observations. Let  $K(l) \subseteq N$  denote the  $l^{\text{th}}$  trader's observation set, i.e. he has access to the data  $y_{K(l)}$  generated by the observations  $Y_{K(l)}$ . Some observations may be shared by many traders, so that the observation sets  $K(l)$  need not be disjoint. As noted above, diverse initial beliefs may induce traders to assign different interpretations to the same data. Public information may be defined as the intersection of all  $K(l)$ .

A trader's information on the environment is not restricted to his observation set alone, because he may also infer from market prices something about data that other traders know but he does not. Such inferences are possible because realized prices depend on the state of nature, and traders are aware of this (possibly through familiarity with the history of the system's behavior). Let  $P$  represent the functional relationship between prices and the environment, i.e.  $p = P(s)$ . Again, the inferences about  $S$  that traders can draw from the price  $p$  have only limited implications on their inferences about the "relevant" aspect  $w$  of the state  $s$ . In recent studies of market information, attention has been devoted to models where all traders' inferences are identical, involving complete knowledge of the "true" relationship between all ultimately observable variables and the price. Their expectations about  $w$  have thus been assumed "self fulfilling", and in some cases this has even been considered a requisite of rational behavior - hence the terminology "fulfilled expectations equilibrium" and "rational expectations equilibrium" associated with many models of markets where prices convey information. In the present study, traders' expectations need not be "self fulfilling". Their inferences about  $w$  may be limited and

heterogeneous, depending on their initial beliefs about  $S$ .

Besides the special role of prices in conveying information, prices naturally also have their usual role of determining each trader's set of possible market positions (demands), by affecting his wealth and the asset combinations that can be attained with this wealth. Let  $X_i(p)$  be the  $i^{\text{th}}$  trader's set of feasible market positions when the price is  $p$ .  $X_i(p)$  reflects the usual budget constraint and any further restrictions that may apply (e.g. short positions, margin requirements, institutional limitations etc.).

Let  $Q_i(p|y_A, P)$  be the  $i^{\text{th}}$  trader's position when the price is  $p$ , given that he knows the data  $y_A$  and given his inferences from the price  $p$ , i.e.

$$Q_i(p|y_A, P) = G_i[X_i(p) | Y_A(s) = y_A, P(s) = p].$$

This notation shows clearly how prices affect the trader's choice both through the feasible set  $X_i(p)$  and through the inferences from  $P(s) = p$ . For given  $y_A$  and  $P$ , the indicated position is a function of  $p$ , say  $D_i(p)$ , mapping ( $J$ -dimensional) prices into ( $J$ -dimensional) demands.<sup>5</sup>  $D_i$  is called the demand schedule or the demand correspondence of the  $i^{\text{th}}$  trader. This demand correspondence is determined by the data he observes and by the inferences that he draws from prices. Let this relationship be denoted by  $L_i$ , i.e.

$$D_i = L_i(y_A | P),$$

which means that when the  $i^{\text{th}}$  trader observes the data  $y_A$  and makes inferences from prices, his demand correspondence is  $D_i = L_i(y_A | P)$ , so that

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<sup>5</sup>Or perhaps into subsets of  $R^J$ .

when the price is  $p$  his position is  $D_i(p)$ , where

$$D_i(p) = Q_i(p; y_A, P).$$

The relationship between the environment, demands, and prices is elaborated upon in the next section.

### III. GENUINE TRADING PROCESSES.

Economic theory has traditionally concentrated on equilibrium considerations (e.g. traders' constrained optimization and market clearing), and studies of the relationship between information and prices are no exception. The usual approach in the analysis of information equilibria is thus to postulate some function  $P$  and investigate its equilibrium properties. Kreps (1975) is careful to state explicitly a basic requirement that is implicitly adopted in such models, i.e. that "...prices (must) contain no more information than is possessed by all (traders)" (p.2).<sup>6</sup> But equilibrium considerations per se - even with an explicit indication of the underlying information - do not make any reference to the way in which available information affects the determination of prices. Indeed, the standard requirement is not inconsistent with a walrasian auctioneer who independently observes all available data and then calls the indicated price, which is indeed verified to be equilibrium. The basic problem here is that equilibrium (and thus also fulfilled expectations equilibrium) does not explicitly indicate the relationship between the participants' trading behavior and

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<sup>6</sup>Formally, this is stated as a requirement that  $P$  be at least as coarse as  $Y_1, \dots, Y_N$ , or equivalently that there exist some function mapping observations into the realized prices.

the realized prices. To capture the essence of our usual understanding of the "market" notion, the model must make some reference to this relationship. We want prices that are generated through traders' behavior. Prices that depend on the environment in a way that cannot be associated with trading behavior may be no more than a spurious property of the model.

A concept of "genuine trading processes" is introduced in this paper to rectify this difficulty. A "trade resolving process"  $M$  is defined as a function mapping I-tuples of demand correspondences into prices, i.e.  $M(D_1, \dots, D_I)$ , written also as  $M(\{D_i\})$ , is the (J-dimensional) price vector when the first trader's demand correspondence is  $D_1$ , the second's is  $D_2$ , etc.<sup>7</sup> We say that the price system is a "genuine trading process" if there exists  $M$  such that

$$P(s) = M(\{L_i[Y_{K(i)}(s); P]\}) \quad \text{for all } s \text{ in } S.$$

In words, a genuine trading process represents a market where prices are fully determined by traders' demands. Two states that give rise to identical demands by all traders must give rise to the same price.<sup>8</sup> We

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<sup>7</sup>To be an equilibrium, a trade resolving process must of course satisfy the requirement that, for any I-tuple of demand correspondences in its domain, if  $p_0 = M(\{D_i\})$  then the sum over  $i$  of all  $D_i(p_0)$  equals the aggregate supply.  $M$  is associated with "genuine" trading in that price outcomes are based on the truly preferred (budget feasible) tentative positions  $D_i(p)$ , and are not manipulable by false statements of these preferences. In a competitive system, the motive for such misrepresentations does not exist, because when each trader believes that his demands will have no more than negligible effect on realized prices he concentrates on getting the most preferable outcome at any given price.

<sup>8</sup>In genuine trading processes  $P$  must be at least as coarse as  $L_1, \dots, L_I$  - clearly a stronger condition than the usually assumed comparison to  $Y_1, \dots, Y_N$ .

can visualize this process as if the market were operating in the following way. After observing the data in his observation set, each trader submits detailed complex limit orders, whereby his transactions in one asset may depend also on prices of other assets. In these limit orders, the desired transactions contingent on any tentative price already reflect all the inferences that he can draw from the fact that this particular price is the equilibrium realization. All orders are entered in "the book", and on the basis of these entries a price is declared. All orders applicable to the declared price are then executed.

#### IV. PRICES THAT FULLY REFLECT INFORMATION.

When inferences about certain information can be derived from prices, it is said that the information is "reflected" in the price. A statement on the information content of prices in a given system must specify two things: (1) what information is reflected in prices, and (2) how prices reflect this information, i.e. what kind of inferences about this information are possible in the given system. These aspects may be considered "dimensions", along which statements on price information can be (partially) ordered. A statement is more far-reaching along the first dimension ("stronger") if it relates to more information, and more far-reaching along the second dimension if it involves more powerful inferences. Of course, in some systems it may be possible to draw far-reaching inferences about some items of data, but only much more limited inferences (or none at all) about larger information sets.

Special interest has been devoted in the literature to the notion of

prices which "fully" reflect some information.<sup>9</sup> The most far-reaching way in which prices may reflect some information is having the information itself fully and accurately inferred from prices. This kind of inference may, however, be more far reaching than the needs of traders. Traders may perhaps be able to draw from prices all the operational implications associated with some information without being able to actually retrieve the data itself. These notions are represented in the following two alternative definitions of systems with price information. In both definitions, the attempt to indicate how prices supposedly "fully reflect" some information involves two aspects: (1) prices provide "satisfactory" inferences, so that recourse to the data itself is unnecessary, and (2) the inferences can be made directly from the price, and they do not depend on the trader's other information or on his preferences.

Definition: The market is fully data-informative with respect to information set  $A \subseteq N$  if the data  $y_A = Y_A(s)$  can be accurately inferred from the price  $p = P(s)$  for all  $s$  in  $S$ .

Definition: The market is fully action-informative with respect to information set  $A \subseteq N$  if the optimal position of any trader, given the data in any information set  $B$ , the data in  $A$ , and the current price is equal to his optimal position given only the data in  $B$  and the price.

The first definition is more far-reaching than the second, as a data-

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<sup>9</sup>In the literature of financial economics, the market is said to be "efficient in the strong form" if prices "fully reflect all available information". It is said to be efficient in the "semistrong" form if this applies only to "the set of all publicly available information", and in the "weak" form if one refers only to the sequence of past prices (cf. Fama 1970).

Informative market is clearly also action-informative with respect to the same information. In both definitions, if the statement applies to some information set  $\bar{A}$ , it also applies to all subsets of  $\bar{A}$ .

#### V. LIMITS OF PRICE INFORMATION.

The theoretical study of models in which prices "fully reflect" some information is concerned primarily with the implications for traders' behavior and with the allocation efficiency of the price system. Empirical investigations seek to determine what kinds of information are actually reflected in observed prices.<sup>10</sup> Both approaches are based on the premise that full inferences from prices may be possible. While some examples indicate that with price information equilibrium may fail to exist, other examples seem to suggest that fully informative prices may be an open possibility. To help determine the relevant domain for both theoretical analysis and empirical work, it is important to identify the possible range of price information. Assuming that equilibrium prices exist, to what extent can they reflect information?

This section sets limits on the feasible extent of price information in genuine trading processes. The possible combinations along the two "dimensions" of price information are severely restricted: only very limited inferences can be made from prices with respect to substantial information sets, and substantial inferences can be made only with respect to very limited information sets. It is shown below that prices cannot

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<sup>10</sup>For a survey of empirical work on price information in the stock market, see Fama (1970) and related papers in Lorie and Brealey (1972).

really "fully reflect" any useful information. At best, they can trivially "reflect" only "inferior" or "basically useless" information. Roughly, inferior information is information that can be accurately inferred from other data, and basically useless information is information that would not affect traders' decisions even if prices did not reflect any part of that information. These terms will be precisely defined presently.

The following notation will sharpen the definitions and help make the exposition precise. For functions  $F_1$  and  $F_2$  defined on  $S$ , we say that  $F_1$  is determined by  $F_2$ , or equivalently that  $F_1$  transforms  $F_2$ , written  $F_1TF_2$ , if there exists a function  $H$  such that

$$F_1(s) = H[F_2(s)] \text{ for all } s \text{ in } S.$$

$F_1TF_2$  is equivalent to " $F_1$  is coarser than  $F_2$ ", i.e. the partition of  $S$  to equivalence classes under  $F_1$  is coarser than the corresponding partition under  $F_2$ . It is evident that  $T$  is transitive and that  $A \subseteq B \subseteq N$  implies  $Y_A TY_B$ . For sets  $A$  and  $B$ , let  $B-A$  denote the set of points in  $B$  and not in  $A$ , and let  $A^* = N-A$  be the complement of  $A$  in  $N$ . Finally, consider  $D_i = L_i[Y_{K(L)}(s)|P]$  as a function on  $S$ , i.e. the state of nature determines, through the observations of the  $i^{\text{th}}$  trader, the demand correspondence that characterizes his behavior. Let  $D$  denote the  $I$ -tuple of demand correspondences of all traders, which will be termed simply "demands". Again, the realized demands depend on the state  $s$ .

Some of our definitions can be neatly restated in terms of this terminology.

- (1) A genuine trading process: prices are determined by demands (PTD).
- (2) The market is fully data-informative with respect to  $A$ : the observations in  $A$  transform prices ( $Y_A TP$ ).



A fully action-informative market can be formally represented as follows:  
 The function  $P$  is fully action-informative with respect to  $A \subseteq N$  when for any  $B \subseteq N$ ,  $y_A$ ,  $y_B$ , and  $p$  which are not mutually inconsistent<sup>11</sup>

$$Q_L(p|y_{A \cup B}, P) = Q_L(p|y_B, P) \text{ for all } l.$$

For simplicity, assume that no information available to any trader is so conclusive as to completely eliminate the possibility of any price in  $P(S)$  (although with some data he may assess the probability of certain prices to be arbitrarily low).<sup>12</sup> The equality must then apply to all  $p$ , and can be written simply as:

$$L_L(y_{A \cup B}|P) = L_L(y_B|P).$$

This indicates that when prices (operationally) reflect the information set  $A$  no demand correspondence depends on the data in  $A$ , hence the following lemma.

Lemma 1: If the market is fully action-informative with respect to  $A$ , then demands are determined by observations not in  $A$  ( $DIY_{A^c}$ ).

Proof: When the market is fully action-informative with respect to  $A$ , it is fully action-informative with respect to  $A^c = A \setminus K(l)$ , because for all  $B$

$$L_L(y_B|P) = L_L(y_{B \cup A}|P) = L_L(y_{B \cup A} \setminus A|P) = L_L(y_{B \setminus A}|P).$$

Hence, with  $B = K(l) - A$

$$L_L(y_{K(l)-A}|P) = L_L(y_{K(l)}|P) \text{ for all } l$$

<sup>11</sup>I.e. the intersection of  $Y_A^{-1}(y_A)$ ,  $Y_B^{-1}(y_B)$  and  $P^{-1}(p)$  is non-empty.

<sup>12</sup>The purpose of this is to preclude "artificial" signaling by informed traders through statements of "irrelevant" demands contingent on prices that they know to be impossible. The same effect could be alternatively attained by assuming that such signaling (prior to taking a position based on their superior information) is contrary to the informed traders' best interests, or by assuming that the market system generates prices that are independent of these "irrelevant" demands and that will therefore not "transmit the signal".

so that  $D_1TY_{K(l)-A}$  and hence  $DTY_{A^*}$ . QED.

Since prices are determined by demands, this leads to the following basic result on the limits of price information.

Theorem 1: If a genuine trading process is fully action-informative with respect to A, prices are determined by observations not in A ( $PTY_{A^*}$ ).

Proof: By transitivity,  $PTD$  and  $DTY_{A^*}$  imply  $PTY_{A^*}$ . QED.

When prices are fully determined by observations not in A, they cannot really "reflect" A. To prove what can be reflected by genuine trading processes, we define the following two characterizations of observation sets.

An observation set A is termed inferior if  $Y_A$  transforms  $Y_{A^*}$ . Data in an inferior observation set can thus be accurately inferred from data not in that set.

An observation set A is termed basically useful if there exist states  $s^1$  and  $s^2$  which involve different preferred positions for some trader, and which are distinguishable only through  $Y_A$ , i.e.

$$\begin{aligned} Y_{A^*}(s^1) &= Y_{A^*}(s^2) = y_{A^*} \\ Y_A(s^1) &= y_A^1 \neq Y_A(s^2) = y_A^2 \\ P(s^1) &= p^1, \quad P(s^2) = p^2 \end{aligned}$$

and for some  $i$  and any function  $F(s)$  such that  $FTY_{A^*}$  also

$$G_i[X_i(p^1); Y_A(s) = y_A^1, F(s) = f] \neq G_i[X_i(p^2); Y_A(s) = y_A^2, F(s) = f].$$

A basically useful observation set will thus, at least under some circumstances, affect some trader's position, regardless of the information he has on observations not in that set. A basically useless

observation set is defined as a set that is not basically useful. It is evident that an inferior set cannot be basically useful,<sup>13</sup> and must therefore be basically useless.

When the observations in A transform prices which are determined by observations not in A, A must be a set of inferior data which merely transforms other observations, i.e.

Theorem 2: If a genuine trading process is fully data-informative with respect to A, then A is inferior.

Proof: A fully data informative market is fully action-informative, hence by theorem 1  $PT_{Y_{A^*}}$ , and by transitivity this and  $Y_{A^*}TP$  imply  $Y_{A^*}TY_{A^*}$ . QED.

If prices are only action informative with respect to A, A need not be inferior. But then there cannot exist two states that induce different demands and that can be distinguished only through  $Y_A$  - because by theorem 1 price is determined by  $Y_{A^*}$  and must be identical for both states, and different demands at this price contradict a fully action-informative market. This is stated in

Theorem 3: If a genuine trading process is fully action-informative with respect to A, A is basically useless.

Proof: Assume to the contrary that A is basically useful. Then, since by theorem 1  $PT_{Y_{A^*}}$ , there exist states  $s^1$  and  $s^2$  such that  $P(s^1)=P(s^2)=p$ ,  $Y_{A^*}(s^1)=Y_{A^*}(s^2)$ , and for some  $i$

$$G_i[X_i(p)|Y_A(s)=y_A^1, P(s)=p] \neq G_i[X_i(p)|Y_A(s)=y_A^2, P(s)=p]$$

or, equivalently

$$Q_i(p|y_A^1, p) \neq Q_i(p|y_A^2, p)$$

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<sup>13</sup>Because the assumed premises cannot hold for an inferior set.

which contradicts  $DTY_{A^*}$  and a fully action-informative market. QED.

Suppose the market is action-informative with respect to A, and some subset B of  $A^*$  transforms A. Then the market must be also action-informative with respect to BUA, which must therefore also be basically useless. This suggests something about the reason why data in a basically useless set A need never be used - even in a non-informative market - if sufficient data from  $A^*$  is available. It cannot be merely because the other data somehow repeats the data in A or contains its equivalent.  $A^*$  must contain some data which is in some sense "superior" to the data in A. This data, say  $A^S$ , cannot be reflected in prices, because then  $AUA^S$  would have to be also basically useless, and so forth.

#### VI. CONCLUDING COMMENTS.

The results of the previous section have an important implication for the theoretical study of market systems. When prices convey information to traders, it is not enough to establish the existence of a hypothetical equilibrium price function. It is also necessary to establish, in addition, the existence of a trade resolving process M which can generate this price function from traders' demands. Indeed, the existence of a trade resolving process is only a first condition for the acceptability of a model as a description of an operating market. Future studies of equilibria in genuine trading processes may well devote some effort to a more detailed characterization of the properties of the trade resolving process. For example, consider two sets of possible demand correspondences,  $\{D_1\}$  and  $\{D_1^*\}$ . Suppose that  $M(\{D_1\})=p_0$ , and suppose

that for all  $I$   $D_I(p) = D_I^*(p)$  on a neighborhood of  $p_0$ . If market clearing is the major consideration in the trade resolving process, a model may be deemed unacceptable if it involves a mapping  $M$  such that  $M(\{D_I^*\}) \neq p_0$ .

A model of a genuine trading process can be made to generate fully informative prices by artificially restricting traders from incorporating the full price information in their submitted demands, or by forcing them to artificially signal their observations to the auctioneer. Technically, this is excluded from our analysis by the basic premise that  $L_I(y_{A \cup B} | P) = L_I(y_B | P)$  whenever prices are fully informative with respect to  $A$ . Models which do not satisfy this premise must interfere with the free incentive compatible behavior of traders. At most, such models can establish the potential of prices to reveal information if, and only if, this potential is at least to some extent ignored by traders.<sup>14</sup>

The stockmarket is often mentioned as an example of a competitive environment where prices may convey information about the stocks' prospects. Kihlstrom and Mirman (1975) have recently pointed to the analogy between "insiders' information" as transmitted by stockmarket prices and the role of consumption goods prices as indicators of quality. In genuine trading processes, price information will never be sufficient to eliminate the trader's need for (direct) reliable information on the

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<sup>14</sup>This paper has been presented in the competitive equilibrium context. Strictly speaking, all the results would also apply to "game" situations, where the "demands"  $I$ -tuple  $D$  can be any equilibrium solution of an  $I$ -person game with limited information, and where the trade-resolving process  $M$  incorporates the operational "rules of the game". However, a recent unpublished communication by Robert Wilson seems to suggest that more sophisticated concepts may be necessary to capture the full scope of price information in the non-competitive framework.

(hidden) quality of neither consumption goods nor financial assets.

Grossman and Stiglitz (1975) have been concerned with the implications of price information for the gathering of costly observations. Within the framework of a special model, they show that if markets are data-informative and information is costly "markets break down", and that zero information cost is a necessary and sufficient condition for fully data-informative prices.<sup>15</sup> Our results indicate that in genuine trading processes even zero information cost is not sufficient for this strong property. Price information in such processes is necessarily limited, and costly observation of basically useful data is economically viable even when prices may reflect some information.

Some frequently encountered types of data are inherently inferior. Typically, this applies to most widely published data, which (for reasons of cost) can be published only in some aggregated form. For example, all summarized balance-sheet data is inferior information, because it merely transforms the data in the basic accounts from which the balance-sheets were derived. The results of this section shed a new light on the implications of price information with respect to this kind of data. If the data may be ignored in a trader's decision-making process, this is not because other traders also have access to the same data (as implied by the notion of "semistrong" stockmarket efficiency), but perhaps because there exists some other information which is superior to this data.<sup>16</sup> This

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<sup>15</sup>Ibid, pages 4 and 46.

<sup>16</sup>How accounting information is "reflected" in stock market prices is a question of major concern in business and financial economics. For a survey of some studies on this issue, see Lorie and Brealey (1972).

superior information is, however, never fully reflected in market prices.

The set of all observations is clearly basically useful. It follows that models in which the market is fully action-informative with respect to the set of all observations (e.g. Grossman 1976, and Grossman and Stiglitz 1975) are inconsistent with a genuine trading process. In the context of the securities' markets, it has been suggested that "buy and hold" would be an optimal investment policy if "all available information is immediately incorporated in prices".<sup>17</sup> Again, this cannot be applicable to a genuine trading process. The main operational implication of this paper is essentially very intuitive. If prices are believed to be derived from traders' demands, each trader is well advised to consider carefully any basically useful information he may have, even if it is widely shared by other traders, and even if prices convey information.

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<sup>17</sup>For example, see Black (1971).

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