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FINANCIAL MARKET EFFICIENCY TESTS

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ABSTRACT

This paper provides a selective survey of the voluminous literature on tests for market efficiency. The ideas discussed include standard autocorrelation tests, multi-period regression tests and volatility tests. The formulation and estimation of models for time-varying volatility are also considered. Dependence in second-order moments plays an important role in implementing and understanding tests for market efficiency. All of the reported test statistics and model estimates are calculated with monthly data on value-weighted NYSE stock prices and dividends. The distributions of the test statistics under various alternatives, including fads and bubbles, are illustrated through the use of Monte Carlo methods. In addition to the standard constant discount rate present value model, we postulate and simulate a new fundamental price relationship that accounts for the time-varying uncertainty in the monthly dividend growth rates. Allowing the discount rate to be a function of the time-varying uncertainty in the dividend process results in a simulated fundamental price series that is broadly consistent with most of the sample statistics of the actual data.

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1. Introduction[*]

Testing for the efficiency of financial markets has generated enormous attention. In this paper we provide a selective survey of the econometric tests and estimation procedures that have been employed in this literature. Throughout the paper we illustrate the different ideas using monthly data on the New York Stock Exchange (NYSE) value-weighted price index and dividend series. Many of the results and techniques apply equally well to other financial markets, however.

As emphasized by Fama (1970, 1991), any test for market efficiency necessarily involves a joint hypothesis regarding the equilibrium expected rate of return and market rationality. The earliest tests for market efficiency were primarily concerned with short-horizon returns, where by short-horizon we refer to holding periods within one year. These tests typically assumed that the expected rate of return was constant through time. It follows that if markets are efficient, the realized returns should be serially uncorrelated. Statistically significant own temporal dependencies at both daily, weekly and monthly frequencies have been documented for a wide variety of different asset categories, but the estimated autocorrelations are typically found to be numerically small. It has been argued that the autocorrelations are spurious or economically insignificant.

At the same time, however, most high frequency financial asset returns cannot be considered independently distributed over time since most returns are characterized by periods of relative tranquility followed by periods of turbulence. Since most modern asset pricing theories involve a direct mean-variance tradeoff, at least at the level of the market return, the explicit

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modeling of time variation in the conditional second-order moments of the returns and the underlying fundamentals process is potentially very important in tests for market efficiency. Of particular importance in these developments has been the ARCH (Autoregressive Conditional Heteroskedastic) and GARCH (Generalized ARCH) time series models. In the absence of any structural model explaining the time-varying second-order moments, simple ARCH models have provided a convenient statistical description of the conditional heteroskedasticity. Bollerslev, Chou and Kroner (1992) provide a recent survey of this literature.

While the short-horizon tests generally suggest only minor violations of market efficiency, defined as constant expected returns, more recent evidence in Fama and French (1988a) and Poterba and Summers (1988), using multi-period regressions and variance ratio statistics, suggests that for longer return horizons a large proportion of the return variance is explainable from the history of past returns alone.[1] Of course, the finding of a large predictable component in long-horizon returns does not necessarily imply market inefficiency, as the variation in expected returns could be due to a time-varying risk premium. Indeed, consistent with the idea of a slowly moving equilibrium risk premium, Fama and French (1988b) find that the variation in dividend yields explains a large proportion of multi-year return predictability. Poterba and Summers (1988), though, argue that the magnitude and variability of the implied risk premium is too large to be explained by appeal to any rational asset pricing theory. They argue that asset prices are characterized by speculative fads in which market prices experience long systematic swings

[1] The estimates in Fama and French (1988a) for monthly U.S. stock returns imply that for three to five year returns up to forty percent of the variability is predictable. The evidence in Kim, Nelson and Startz (1991) suggests that much of this predictability may be driven by data around the time of the Great Depression.

away from rational fundamentals prices. These highly serially correlated fads are difficult to distinguish from a martingale model on the basis of the earlier short-horizon tests for market efficiency, but their existence is more evident in long-horizon autocorrelations of returns.

Subsequent work has illustrated that the apparent predictability of the long-horizon returns should be interpreted very carefully. There are very few degrees of freedom, and the overlapping nature of the data in the multi-year return regressions gives rise to a non-standard small sample distribution of the test statistics, which appear to be better approximated by the alternative asymptotic distribution derived by Richardson and Stock (1989). The overlapping data problem may also be overcome by using the vector autoregressive techniques discussed in Baillie (1989) and Hodrick (1992).[2] Interestingly, while the own long-horizon return predictability may be spurious, the statistically more reliable return forecasting specifications employed in Campbell (1991), Hodrick (1992) and Bekaert and Hodrick (1992) suggest a statistically and economically significant long-horizon return predictability on the basis of fundamental variables including dividend yields and interest rates.

The variance bounds or volatility tests pioneered by Shiller (1979, 1981) and LeRoy and Porter (1981) constitute another important class of tests for market efficiency. In the first generation of volatility tests the null hypothesis was taken to be the standard present value model with a constant discount rate. The vast majority of these tests resulted in apparent clear rejections of market efficiency, with actual asset prices being excessively volatile compared to the implied price series calculated from the discounted value of the expected or actual future fundamentals. One possible explanation for this finding was the idea that asset prices may be characterized by self-

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[2] We do not discuss these techniques here because of related coverage in this volume.

fulfilling speculative bubbles that earn the fair rate of return, but cause prices to differ from their rational fundamentals. Flood and Hodrick (1990) provide a survey of this literature.

Although some economists initially viewed volatility tests differently from traditional autocorrelation-based tests of market efficiency, volatility tests are equivalent to standard Euler-equation based tests in the sense that each involves a joint hypothesis regarding the return generation process and the first order condition for economic agents. Cochrane (1991a) provides a recent discussion of this position. In fact, relaxing the assumption of a constant discount rate results in much more mixed conclusions regarding excess volatility and market inefficiency. Additionally, many of the early volatility tests did not take seriously the non-stationarity of prices and fundamentals in calculating and interpreting the test statistics. At the same time, this non-stationarity gives rise to a robust testable cointegrating relationship based on the present value model that remains valid in the presence of stationary stochastic discount rates.

In summary, the current challenges for asset pricing theories can be expressed as the search for a model of expected return variability that is consistent with the empirical findings pertaining to the predictability of returns and that provides an explanation for the pronounced volatility clustering in returns. In this survey, we illustrate how a present value model for the NYSE price index that accounts for the time-varying uncertainty in dividend growth rates can actually explain most of the rejections of market efficiency on the basis of the different tests discussed above. In particular, the conditional mean and variance of monthly NYSE dividend growth rates both have a distinct seasonal pattern, whereas annual dividend growth rates show little serial correlation and appear homoskedastic. Using simulation methods, we show how incorporating this predictable monthly time variation into a model with stochastic discount rates provides a reconciliation of the actual empirical findings in tests for market efficiency with the present value relationship. In addition to this new fundamental price process, we also report simulations for the conventional constant discount rate present value model, together with fads and bubble alternatives. The simulation approach employed in this paper was inspired by the earlier work of Mattey and Meese (1986) which could be usefully read in conjunction with the present study.

The plan for the rest of the paper is as follows. The next section presents the different simulated models used throughout the paper along with a discussion of the seasonal ARCH model for the fundamental dividend process. Section 3 reports a number of short-horizon summary statistics for the NYSE return series and illustrates how the volatility clustering in the returns may be conveniently modelled with a GARCH formulation for the conditional variance. Section 4 examines the long-horizon return tests. While the multi-period regression test statistics may be severely biased in small samples, we show to develop tests based on an iterated version of the null hypothesis using Hansen's (1982) GMM (Generalized Method of Moments). We find some improvement in the small sample performance of the test statistics using this method. In section 5, tests for market efficiency based on the ideas underlying cointegration are briefly analyzed. Section 6 considers a recent class of volatility tests derived by Mankiw, Romer, and Shapiro (1991) and examines the relation of excess volatility to expected return variability. Section 7 provided some concluding remarks.

2. *Data Generation Mechanisms*

In this section we describe the different data generation mechanisms used below in the Monte Carlo experiments. According to the standard present value relationship, the fundamental real price of an asset at time t equals

$$P_t^f \equiv E_t\left[\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}\rho_{t+i}\right)D_{t+j}\right], \tag{2.1}$$

where $E_t[\cdot]$ denotes the mathematical expectation conditional on all information available at time t, and D_t refers to the accumulated real dividend or other payoff on the asset from time t-1 through t.[3] Finally, the discount factor is

$$\rho_t \equiv \exp(-r_t), \tag{2.2}$$

where r_t is the continuously compounded required rate of return. Derivation of equation (2.1) imposes a transversality condition that the market fundamental price does not grow faster than the expected value of the product of the discount factors. If the present value model is true and markets are efficient, the observed price process should equal the fundamental price in equation (2.1) with the required rate of return being driven by a risk premium. This fundamental price relationship, coupled with an explicit formulation for the discount rate, r_t, forms the basis for three of our simulated models, considered below. Two alternative simulations involve explicit deviations of the actual price process, P_t, from the fundamentals price, P_t^f.

Of course, simulation of any of these price processes requires a characterization of the stochastic process governing dividends. All of the

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[3] Although we illustrate the tests for market efficiency using stock market data, the same ideas apply equally well to tests of other present value relations such as the rational expectations theory of the term structure of interest rates; see Campbell and Shiller (1987), for example. We also note that dividends are not the only way of distributing value to shareholders, as Bagwell and Shoven (1989) document, and that dividends are a decision variable of management which may choose to smooth them or to have a liquidating dividend. Marsh and Merton (1986) note that such behavior can cause serious problems with econometric analysis of present value relations.

simulations are based on a model for an annualized dividend series, defined as the current value of the monthly dividends over the previous year,

$$D_t = \sum_{j=0}^{11}(ND_{t-j}/CPI_t)\prod_{h=1}^{j}(1+i_{t-h+1}), \tag{2.3}$$

where the product from h = 1 to 0 is defined to be one. In equation (2.3) i_t denotes the monthly U.S. Treasury bill rate at the beginning of month t, ND_t is the nominal value-weighted NYSE dividend series during month t, and CPI_t is the corresponding monthly U.S. consumer price deflator. With monthly data from January 1926 through December 1987, there are 733 observations on D_t starting in December 1926. We chose to work with the annualized dividend series in equation (2.3), as opposed to ND_t/CPI_t, to reduce the magnitude of the seasonality. This same dividend series is used in Hodrick (1992). Plots of the logarithmic dividend series, $d_t \equiv \ln(D_t)$, together with the logarithmic value-weighted, CPI-deflated, NYSE price index, $p_t \equiv \ln(P_t)$, are given in Figure 1.

It is apparent from Figure 1 that both dividends and prices experienced growth in real terms during the sample period. In the subsequent analysis we therefore concentrate on modelling the growth rate in the dividend process, i.e. $\Delta d_t \equiv d_t - d_{t-1}$. Formal augmented Dickey and Fuller (1981) tests for a unit root in the autoregressive polynomial in the univariate time series representation for d_t support the idea that the dividend growth rate is stationary. In particular, on running the regression,

$$\Delta d_t = \mu + \gamma d_{t-1} + \phi_1 \Delta d_{t-1} + \ldots + \phi_{12} \Delta d_{t-12} + \varepsilon_t, \tag{2.4}$$

the t-statistic for $\gamma = 0$ equals -1.35, far above the asymptotic one-sided one and five percent critical values of -3.43 and -2.86, respectively. We shall return to this and other tests for unit roots in both dividends and prices in our discussion of cointegration based tests for market efficiency in Section 5.

Examination of the correlation structure for Δd_t indicates a distinct seasonal pattern with highly significant autocorrelations at the seasonal frequencies and a clear cutoff in the partial autocorrelation function at lag twelve.[4] This is consistent with the well-known observation that dividends are lumpy with payoffs concentrated at certain times of each quarter. To capture the seasonal dependence in the annual dividend series we estimate an unrestricted AR(12) model for Δd_t as in equation (2.4) with $\gamma = 0$. It is certainly possible that a seasonal ARMA model might provide a more parsimonious representation, but as a data generating process, the unrestricted AR(12) model conveniently captures the own temporal dependencies in the conditional mean of the dividend series.

The residuals from this AR(12) model are uncorrelated, but they are clearly not independently distributed through time. Strong seasonality remains in the uncertainty associated with dividend growth, as manifest by highly significant autocorrelations of the squared residuals at the seasonal lags.[5] To account for this feature of the dividend process, we estimated a restricted ARCH(12) model for the conditional variance. ARCH models were first introduced by Engle (1982) and have subsequently found very wide use in the modelling of volatility clustering in high frequency financial data. This is discussed further in Section 3 below. The maximum likelihood estimates for the AR(12)-ARCH(12) model for Δd_t obtained under the assumption of

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[4] Even though the annualized dividend series exhibit seasonality, it is worth noting that the degree of own temporal dependence is substantially reduced when compared to the real monthly growth rate in the raw dividend series. For instance, the Ljung and Box (1978) portmanteau test statistic, defined formally below, for up to twelfth order serial correlation in Δd_t equals 342.7 compared to 4042.7 for $\Delta \ln(ND_t)$.

[5] The autocorrelations for the squared residuals at lags 1, 3, 6 and 12 equal 0.066, 0.245, 0.120 and 0.164 respectively, and all but lag one exceed the five percent critical of $1.96/\sqrt{T} = 0.073$ under the null of $\hat{\varepsilon}_t^2$ i.i.d. through time.

conditional normality for the sample period January 1928 through December 1987 are

$$\Delta d_t = \underset{(0.00038)}{0.00033} - \underset{(0.048)}{0.035}\,\Delta d_{t-1} + \underset{(0.027)}{0.053}\,\Delta d_{t-2} + \underset{(0.062)}{0.337}\,\Delta d_{t-3} + \underset{(0.032)}{0.065}\,\Delta d_{t-4}$$

$$+ \underset{(0.023)}{0.018}\,\Delta d_{t-5} + \underset{(0.061)}{0.121}\,\Delta d_{t-6} + \underset{(0.026)}{0.010}\,\Delta d_{t-7} - \underset{(0.021)}{0.019}\,\Delta d_{t-8} + \underset{(0.036)}{0.182}\,\Delta d_{t-9}$$

$$- \underset{(0.026)}{0.009}\,\Delta d_{t-10} + \underset{(0.021)}{0.012}\,\Delta d_{t-11} - \underset{(0.058)}{0.238}\,\Delta d_{t-12} + \varepsilon_t \tag{2.5}$$

$$\varepsilon_t \,|\, I_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \underset{(0.000008)}{0.000042} + \underset{(0.078)}{0.105}\,\varepsilon_{t-1}^2 + \underset{(0.075)}{0.289}\,\varepsilon_{t-3}^2 + \underset{(0.085)}{0.164}\,\varepsilon_{t-6}^2 + \underset{(0.113)}{0.287}\,\varepsilon_{t-12}^2.$$

The notation I_{t-1} refers to the information set consisting of the past history of the dividend process. Robust asymptotic standard errors as in Bollerslev and Wooldridge (1992), which are discussed in Section 3, are reported in parentheses. Notice that the seasonal AR and ARCH coefficients are all statistically important. Also, standard summary statistics, available upon request, indicate that this relatively simple time series model provides a good description of the own temporal dependencies in Δd_t.[6]

Of course, the fundamental price series defined in equation (2.1) depends on forecasts of the levels of the future dividends, D_t, as opposed to the growth rates modeled in equation (2.5). Using a first order Taylor series expansion, it follows that

$$E_t(D_{t+j}) \approx \exp[E_t(d_{t+j}) + 0.5\,Var_t(d_{t+j})], \quad j = 1, 2, \ldots . \tag{2.6}$$

Equation (2.6) is satisfied exactly if the dividend growth rate is conditionally normally distributed. Closed-form expressions for $E_t(d_{t+j})$ and $Var_t(d_{t+j})$ are available by expressing the AR(12)-ARCH(12) model in first-order companion form, as in Baillie and Bollerslev (1992). These expressions are presented in the Appendix.

In practice, the infinite sum in equation (2.1) is necessarily truncated at some value J.[7] Since the process for dividends is difference stationary, it follows that for large values of j,

[6] The maximized value of the conditional normal log-likelihood function for the model in equation (2.5) equals -4472.4 compared to -4644.8 for the homoskedastic normal AR(12) model restricting the four ARCH coefficients to be zero. The resulting likelihood ratio test statistic for no ARCH equals 344.8, which is highly significant.

[7] While a closed form expression for the infinite discounted sum in equation (2.1) may be derived using the methods of Hansen and Sargent (1980, 1981) in the case of a constant discount rate, see e.g., West (1987), the presence of time-varying discount rates coupled with time-varying conditional variances renders a closed form solution infeasible.

$$E_t(d_{t+j+1}) = E_t(d_{t+j}) + \eta, \qquad\qquad j = J,\ J+1,\ \ldots, \qquad (2.7)$$
$$Var_t(d_{t+j+1}) = Var_t(d_{t+j}) + \delta,$$

where $\eta = E(\Delta d_t)$ denotes the unconditional expected real growth rate, and δ denotes the unconditional increase in the prediction error uncertainty at long horizons. For the model estimates reported in equation (2.5), $\eta = 0.00066$ and $\delta = 0.00106$.[8] Also, suppose that the expected future discount rate associated with distant dividends is approximately constant, then

$$E_t(\rho_{t+j+1}) = \rho = \exp(-r), \qquad\qquad j = J,\ J+1,\ \ldots . \qquad (2.8)$$

In the implementation, we took J = 120, corresponding to a forecast horizon of 10 years. Some informal sensitivity analysis revealed almost no change in the results with a longer truncation lag. Combining equations (2.1), (2.6), (2.7) and (2.8), the fundamental price may be approximated by

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[8] Let $\zeta(L) = \phi(L)^{-1}$ denote the lag polynomial in the infinite moving average representation for the AR(12) model with parameters ζ_i, where $\phi(L)\Delta d_t = \mu + \varepsilon_t$. It follows that

$$Var_t(d_{t+j+1}) = Var_t(d_{t+j}) + \sum_{h=0}^{j} \sum_{i=0}^{h} \zeta_h \zeta_{h-i} E_t(\varepsilon^2_{t+j+1-h}).$$

Thus, in the limit

$$\delta = \sigma^2 \sum_{h=0}^{\infty} \sum_{i=0}^{h} \zeta_h \zeta_{h-i},$$

where σ^2 equals the unconditional variance of ε_t. Also, $\eta = \mu\phi(1)^{-1}$.

$$P_t^f = \sum_{j=1}^{J-1}\left(\prod_{i=1}^{j} E_t(\rho_{t+i})\right)\exp\left(E_t(d_{t+j}) + 0.5\,\mathrm{Var}_t(d_{t+j})\right)$$

$$+ \sum_{j=J}^{\infty}\left(\rho^{j-J}\prod_{i=1}^{J} E_t(\rho_{t+i})\right)\exp\left(E_t(d_{t+J}) + 0.5\,\mathrm{Var}_t(d_{t+J})\right)\exp\left((\eta + 0.5\delta)(j-J)\right)$$

$$= \sum_{j=1}^{J-1}\left(\prod_{i=1}^{j} E_t(\rho_{t+i})\right)\exp\left(E_t(d_{t+j}) + 0.5\,\mathrm{Var}_t(d_{t+j})\right)$$

$$+ \left(\prod_{i=1}^{J} E_t(\rho_{t+i})\right)\exp\left(E_t(d_{t+J}) + 0.5\,\mathrm{Var}_t(d_{t+J})\right)\sum_{j=0}^{\infty}\left(\rho\exp(\eta + 0.5\delta)\right)^{j} \quad (2.9)$$

$$= \sum_{j=1}^{J-1}\left(\prod_{i=1}^{j} E_t(\rho_{t+i})\right)\exp\left(E_t(d_{t+j}) + 0.5\,\mathrm{Var}_t(d_{t+j})\right)$$

$$+ \left(\prod_{i=1}^{J} E_t(\rho_{t+i})\right)\exp\left(E_t(d_{t+J}) + 0.5\,\mathrm{Var}_t(d_{t+J})\right)\left(1 - \rho\exp(\eta + 0.5\delta)\right)^{-1},$$

where for the last equality to hold true it is assumed that $\rho^{-1} = \exp(r) > \exp(\eta + 0.5\delta)$. Note that a sufficient condition for the validity of the approximation in equation (2.9) is that the expected future discount rates and dividend growth rates are conditionally uncorrelated which is trivially satisfied with a constant discount rate.

We simulate five different price series using the present value relationship in equation (2.9) and the estimated model for the dividend growth rate in equation (2.5). The Null1 prices are calculated under the assumption of a constant discount rate,

$$E_t(\rho_{t+j}) = \rho = \exp(-r), \qquad j = 1, 2, ..., \tag{2.10}$$

and no ARCH effects in the dividend series; i.e. ε_t i.i.d. normally distributed with a variance equal to the implied unconditional variance from equation (2.5). The discount rate is set at $r = 0.00635$, corresponding to the 7.9 percent real annual return on the NYSE value-weighted index over the sample period underlying the estimation results for the dividend model in equation (2.5). The Null1 model is the standard present value relationship typically employed in volatility tests, most of which use annual data. The use of monthly data provides a richer representation of the data generation process and possibly avoids serious temporal aggregation bias.

Following Shiller (1984), Summers (1986), and Poterba and Summers (1988) we also consider a Fads model as a possible explanation for the actual empirical findings. According to the Fads alternative, the market price differs from the fundamental price by a highly serially correlated fad. More formally, let P_t^f denote the fundamental price under Null1 as described above. The Fads price series is then generated according to

$$P_t = \exp\left(\ln(P_t^f) + \ln(F_t)\right), \tag{2.11}$$

where $\ln(F_t)$ follows the AR(1) process,

$$\ln(F_t) = \phi \ln(F_{t-1}) + u_t, \tag{2.12}$$

with u_t i.i.d. normally distributed. In the simulations, $\phi = 0.98$, and σ_u^2 equals 0.330 times the unconditional variance of $\Delta\ln(P_t^f)$ for the particular sample realization. With these parameter choices the process for the fad accounts for twenty-five percent of the unconditional variance in the change in logarithmic price.[9]

In the Bubble alternative, the price differs from the Null1 present value relationship by a self-fulfilling speculative bubble. That is,

$$P_t = P_t^f + B_t, \tag{2.13}$$

where B_t earns the required real rate of return, r,

$$E_{t-1}(B_t) = B_{t-1}\exp(r). \tag{2.14}$$

Notice that the existence of such bubbles violates the transversality condition underlying the fundamental pricing condition in equation (2.1). Stochastic collapsing bubbles were introduced by Blanchard and Watson (1982). The bubble continues with a certain conditional probability and collapses otherwise, where the probability weighted average of these two events must satisfy equation (2.14). The bubble simulated here takes the form,

$$\begin{aligned} B_t = {} & I_{z_t \le \pi_{t-1}}(\pi_{t-1})^{-1}\left[\exp(r)B_{t-1} - (1-\pi_{t-1})(0.1P_{t-1}^f)\right]\exp(v_t - 0.5\sigma_v^2) \\ & + I_{z_t > \pi_{t-1}}(0.1P_{t-1}^f). \end{aligned} \tag{2.15}$$

In equation (2.15) the probability of the bubble continuing is denoted π_{t-1}. If the bubble bursts, it does not collapse to zero but begins again at a value of 0.1

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[9] From equations (2.11) and (2.12) it follows that, $Var[\Delta ln(P_t)] = Var[\Delta ln(P_t^f)] + Var[\Delta ln(F_t)] = Var[\Delta ln(P_t^f)] + 2\sigma_u^2(1+\phi)^{-1}$. Setting $\phi = 0.98$ and requiring that $2\sigma_u^2(1+\phi)^{-1}$ equals twenty-five percent of $Var[\Delta ln(P_t)]$ implies $\sigma_u^2 = 0.330 Var[\Delta ln(P_t^f)]$.

times the fundamental price, P_t^f.[10] To generate stochastic bubbles we drew an i.i.d. random variable z_t from the uniform distribution on the unit interval. The indicator variable $I_{z_t \leq \pi_{t-1}}$ signifies whether $z_t \leq \pi_{t-1}$. If the bubble continues, an innovation in the bubble is generated from v_t. By assumption, v_t is i.i.d. normally distributed with mean zero and variance σ_v^2, so that $E_{t-1}[\exp(v_t - 0.5\sigma_v^2)] = 1$. We allow the probability that the bubble will collapse to depend explicitly on the current deviation from the fundamental price,

$$1 - \pi_t = 2\left[1 - \Phi(P_t^f/B_t)\right], \tag{2.16}$$

where $\Phi(\cdot)$ refers to the cumulative standard normal distribution function. The larger the bubble relative to the fundamental price, the greater the chance of a collapse. In the simulations we set $\sigma_v^2 = 0.0009$. This parameterization of the bubble process led to an average of eight collapsing bubbles, a minimum of two and a maximum of sixteen, across the 1000 simulations of 720 months of data.[11]

The Null2 model incorporates the serial dependence in the conditional variances into the optimal forecasts for the levels of future dividends using the forecast formula for the conditional variances based on model (2.5). Details of these calculations are in the Appendix. Given the strong conditional heteroskedasticity in the data, we think it is important in simulations to

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[10] Our stochastic bubbles do not collapse to zero because as Diba and Grossman (1988) note, the theoretical impossibility of a rational negative bubble rules out a zero-mean innovation in a bubble starting at zero. Hence, the bubble would have to be always in the stock price.

[11] We also imposed two other restrictions on the bubble process. We required that the bubble continue with probability one if its current value is less than the reversion value under a collapse, and we set the maximum probability of collapse at 0.99. Flood and Hodrick (1986, 1990) provide a discussion of tests for bubbles.

explicitly account for higher order moment dependencies in the fundamentals in a pricing relationship as illustrated in equation (2.9). In accordance with the Null1, Fads and Bubble series, the Null2 alternative maintains the assumption of a constant discount rate, r = 0.00635.

The final model develops a crude time-varying risk premium, or TVRP, price series. It extends the Null2 alternative by allowing for a variable discount rate. In order to keep things relatively simple and to avoid the use of additional data series, we do not work directly with any formal structural model of the variable discount rate. Instead, we simply postulate that the discount rate is a linearly increasing function of the change in the prediction error uncertainty associated with future values of the fundamental dividend process. Specifically, in equation (2.2) we set

$$r_{t+i} = \lambda[\mathrm{Var}_t(d_{t+i}) - \mathrm{Var}_t(d_{t+i-1})] \qquad i=1,2,\ldots, \tag{2.17}$$

where $\lambda = 6.017$. This choice of λ ensures that the unconditional discount rate associated with long-horizon predictions converges to the sample real returns employed in the other simulations.

3. *Short-Horizon Returns*

In this section we review some of the time series techniques and test statistics used to examine the short-run temporal dependencies in asset returns and their relation to the market efficiency hypothesis. It is generally accepted that most high frequency returns are approximately linearly unpredictable, although this is not a requirement of an efficient market. It is also well recognized that returns are characterized by volatility clustering and leptokurtic unconditional distributions. The documentation of these facts dates back to at least Mandelbrot (1963) and Fama (1965).

The first column in Table 1 reports a number of summary statistics for the real monthly value-weighted NYSE percentage rates of return for the sample period January 1928 through December 1987. These returns are denoted by R_t throughout the paper. As noted in the previous section, the average real monthly return on the index over this period was 0.635 percent, or 7.9 percent on an annual basis. The realized return is very variable around this mean return, however, with a monthly variance of 33.5. Multiplying this monthly variance by 12, as if the returns are serially uncorrelated, produces an annualized standard deviation of 20.1 percent. This high degree of return variability is also obvious from the plot in Figure 2.

The sample skewness and kurtosis coefficients are reported in the next two rows of Table 1.[12] The column labelled Asymp. provides p-values under the null hypothesis of i.i.d. normally distributed returns, i.e. $b_3 = 0$ and $b_4 = 3$. The median values in the asymptotic distributions are reported in square brackets.[13] The results indicate some positive skewness and very pronounced leptokurtosis in the sample unconditional distribution of returns.

The first-order sample autocorrelation coefficient is denoted by ρ_1. Although statistically significant, the estimated value implies that only 1.3 percent of the total variation in the return is explainable from last month's return alone. Furthermore, the predictability of this index return may, in part, be attributed to a non-synchronous trading phenomenon, as discussed in Lo and MacKinlay (1988) and Fama (1991). The next row in Table 1 reports the Ljung and Box (1978) portmanteau test for up to twelfth order serial correlation, Q_{12}.[14] The second column indicates that this test statistic is highly

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[12] Let $\hat{\mu}_i$ denote the i-th centered sample moment. Then, $b_3 \equiv \hat{\mu}_3/(\hat{\mu}_2)^{3/2}$, and $b_4 \equiv \hat{\mu}_4/(\hat{\mu}_2)^2$.

[13] Under the null of i.i.d. the standardized test statistics, $\sqrt{(T/6)}b_3$ and $\sqrt{(T/24)}(b_4-3)$, should both be the realization of a standard normal distribution; see e.g. Jarque and Bera (1980).

[14] Let ρ_i denote the i-th sample autocorrelation. The Ljung and Box (1978) test for up to N-th order serial dependence is then given by $Q_N = (T+2)T[\rho_1^2/(T-1) + \rho_2^2/(T-2) + ... + \rho_N^2/(T-N)]$ where T denotes the sample size.

significant at conventional levels in the asymptotic chi-square distribution under the null hypothesis of i.i.d. observations. As noted by Diebold (1986) and Cumby and Huizinga (1992), however, the presence of conditional heteroskedasticity or excess kurtosis will bias the portmanteau test towards over rejection of the less restrictive null hypothesis of uncorrelated returns. Also, excluding the first lag, the test for the joint significance of lags two through twelve equals only 22.6.

Recently, a number of authors have also employed so-called variance ratio statistics,

$$v(k) \equiv \mathrm{Var}(R_{t+k-1} + R_{t+k-2} + \ldots + R_t)/[k\,\mathrm{Var}(R_t)], \qquad (3.1)$$

as an alternative way of summarizing own temporal dependencies, particularly at horizons longer than one year. If the returns at horizon k are dominated by positive autocorrelation, the variance ratio is greater than one, whereas predominantly negative autocorrelation results in a variance ratio below one.[15] Consistent with the results reported in Lo and MacKinlay (1988), the variance ratio at the one-year horizon for the real value-weighted NYSE returns equals

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Under the null hypothesis of i.i.d. observations, Q_N has an asymptotic chi-squared distribution with N degrees of freedom.

[15] The variance ratio statistic in equation (3.1) is consistently estimated by $\hat{v}(k) = \{1+2[(k-1)\rho_1 + (k-2)\rho_2 + \ldots + \rho_{k-1}]\}/k$. Under the null hypothesis of i.i.d. observations, with k = 12 and T = 720, the standard error for the variance ratio statistic in the asymptotic normal distribution with mean one equals 0.140. A heteroskedasticity consistent standard error may be calculated from White's (1980) covariance matrix estimator for the sample autocorrelations; see Lo and MacKinley (1988) for further details.

1.151. This indicates only minor and statistically insignificant positive short-run own dependencies in returns. We defer discussion of longer-run dependencies to Section 4.

In contrast to the weak evidence for autocorrelation in returns, the last two rows in Table 1 highlight the importance of conditional heteroskedasticity. The first-order autocorrelation coefficient for the squared returns, $\rho_1^{(2)}$, and the portmanteau test for up to twelfth-order serial correlation in the squared returns, $Q_{12}^{(2)}$, are highly significant at any reasonable level. The variance ratio statistic for the squared returns equals 3.027 at the one year horizon, indicating very significant positive dependence. This pronounced dependence in the second-order moments is immediately evident from Figure 2. It is worth noting that this finding of strong dependence in the even ordered moments does not necessarily imply market inefficiency. The presence of high-frequency volatility clustering is perfectly consistent with a martingale hypothesis for stock prices which is implied by the assumption of a constant short-run expected rate of return.[16] In addition, though, modern asset pricing theories rely crucially on time-variation in the second-order moments of returns and market fundamentals as sources of rational time-varying risk premia as in Abel (1988), Hodrick (1989), and Bekaert (1992). We return to this issue in more detail below.

[16] The martingale model is sometimes incorrectly referred to as the random walk model. Whereas the random walk model assumes i.i.d. innovations, a martingale difference sequence only stipulates that the innovations be serially uncorrelated, or white noise.

We now analyze the simulation results for the various price processes discussed in Section 2 as possible explanations of these empirical findings. The last five columns of Table 1 report the p-values for the sample statistics under the different data generating mechanisms along with the median values of the statistics in the simulated distributions over the one thousand replications. If the sample statistic does not fall within the empirical distribution generated by a particular model, we will judge the model as being inconsistent with the data along that dimension.

The Null1 constant discount rate model with no ARCH effects and the Fads alternative are both unable to explain the magnitude of the unconditional variance of returns. The medians of the Null2 and Bubble distributions are also considerably lower than the sample statistic of 33.5, although the distributions from these models are more disperse. Only for the TVRP alternative is the p-value relatively large, 0.282, and the median of the simulated distribution close to the sample unconditional one-month return variance. Similarly, neither the Null1 nor the Fads alternative with normally distributed errors is able to explain the non-normality of returns. In contrast, the Bubble specification results in a considerably higher median kurtosis coefficient than the sample analogue. The simulated Null2 and TVRP alternatives are both consistent with the actual data regarding the sample unconditional kurtosis.

None of the five models produces any positive first-order autocorrelation in the medians of the test statistics. But, both the Null2 and TVRP alternatives lead to test statistics for the joint significance of the first twelve autocorrelations, Q_{12}, that are broadly consistent with the actual data.

As noted above, a striking feature of the monthly returns is volatility clustering. Although the portmanteau statistic for the first twelve autocorrelations of the squared returns is inconsistent with the null hypothesis of i.i.d. observations in the asymptotic distribution and with the simulated Null1, Fads and Bubbles models, the Null2 and TVRP price series yield test statistics that exceed the sample value of 398.4 in 6.4 and 13.9 percent of the replications, respectively.

In order to better understand the nature of this volatility clustering, we follow Bollerslev (1986, 1987) and estimate an MA(1)-GARCH(1,1) model for the monthly real percentage rates of return:

$$R_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t$$

$$E_{t-1}(\varepsilon_t) = 0, \tag{3.2}$$

$$E_{t-1}(\varepsilon_t^2) \equiv \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2.$$

This relatively simple non-linear time series model provides a useful characterization of the temporal variation in the second-order moments of returns for a wide variety of financial assets.

A number of alternative estimation schemes are available for the model in equation (3.2). The estimation results reported below are all obtained under the auxiliary distributional assumption of conditional normality; that is $\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$, where I_{t-1} refers to the information set at time t-1. In particular, if $\xi' \equiv (\mu, \theta, \omega, \alpha, \beta)$ denotes the vector of unknown parameters, it follows by a standard prediction error decomposition argument that conditional on the initial observations, the quasi-log-likelihood function for the sample realizations $\{R_1, R_2, \cdots, R_T\}$ takes the form

$$L_T(\xi) = \sum_{t=1}^{T} l_t(\xi) = \sum_{t=1}^{T} -0.5\left(\ln(\pi) + \ln(\sigma_t^2) + \varepsilon_t^2\sigma_t^{-2}\right). \tag{3.3}$$

While standard maximum likelihood theory requires the correct distributional assumptions, asymptotic standard errors that remain valid in the absence of conditional normality may be calculated from the matrix of the outer products of the gradients post and pre-multiplied by an estimate of the Hessian; see e.g., Domowitz and White (1982) and Weiss (1986). A simple expression for this estimator in the context of dynamic models with conditional heteroskedasticity that involves only first derivatives is given in Bollerslev and Wooldridge (1992).[17] The quasi-maximum-likelihood estimates, obtained by maximizing equation (3.3) with robust standard errors in parentheses are given in the first column of Table 2.

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[17] Let μ_t and σ_t^2 denote the conditional mean and variance functions, with gradients $\nabla\mu_t$ and $\nabla\sigma_t^2$, respectively. The asymptotic covariance matrix for the quasi-maximum likelihood estimator, $\hat{\xi}_T$, is then consistently estimated by $A_T B_T^{-1} A_T$, where

$$A_T = \Sigma_{t=1}^{T} \nabla\mu_t(\nabla\mu_t)'\sigma_t^{-2} + 0.5\nabla\sigma_t^2(\nabla\sigma_t^2)'\sigma_t^{-4},$$

$$B_T = \Sigma_{t=1}^{T} \nabla l_t(\xi)\nabla l_t(\xi)',$$

and

$$\nabla l_t(\xi) = \nabla\mu_t\sigma_t^{-2}\varepsilon_t + 0.5\nabla\sigma_t^2\sigma_t^{-4}(\varepsilon_t^2 - \sigma_t^2),$$

all evaluated at $\hat{\xi}_T$.

The significant MA(1) term in the conditional mean captures most of the autocorrelation in the returns. The Q_{12} portmanteau test for significant autocorrelations within a year drops from 32.3 for the raw data to 14.2 for the standardized residuals, $\hat{\varepsilon}_t\hat{\sigma}_t^{-1}$, from the model in (3.2). The estimate for $\alpha + \beta$ indicates a very long memory in the conditional variance. The implied half life of a shock to the conditional variance equals $\ln(1/2)/\ln(\hat{\alpha} + \hat{\beta}) = 43.0$ months. This high degree of persistence corresponds to the findings for a large number of other financial assets as noted by Bollerslev and Engle (1992). The parameterization for the conditional variance in equation (3.2) does a very good job of tracking the strong temporal dependence in the variance.[18] The $Q_{12}^{(2)}$ portmanteau test statistic for the squared standardized residuals, $\hat{\varepsilon}_t^2\hat{\sigma}_t^{-2}$, equals 11.8 compared to 398.4 for the raw squared returns.

The negative skewness coefficient for $\hat{\varepsilon}_t\hat{\sigma}_t^{-1}$ in Table 2 contrasts sharply with the positive value for b_3 reported in Table 1. Negative skewness is consistent with the so-called leverage effect in which volatility increases with bad news but decreases with good news as analyzed by Black (1976) and Christie (1982). This observation also provides one of the primary motivations

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[18] A possible explanation for this phenomenon and the estimate of $\alpha + \beta$ close to one is provided by the continuous time approximation arguments given in Nelson (1992) and Nelson and Foster (1991). The apparent strong persistence in the conditional variance could also be a result of stochastic regime changes as in the analysis of Cai (1992).

behind the Exponential GARCH model in Nelson (1991) that allows both the sign and the magnitude of past shocks to influence the conditional variance.[19]

Even though the GARCH(1,1) model does a very good job of capturing the dependence in the second-order moments, the temporal variation in the conditional variance does not explain all the leptokurtosis in the data. Again, this is not unique to the present return series. As an alternative to the robust quasi-maximum-likelihood procedures, the conditional distribution for $\varepsilon_t\sigma_t^{-1}$ could be parameterized directly or estimated by non-parametric methods, as discussed by Bollerslev (1987), Gallant and Tauchen (1989), and Engle and Gonzales-Rivera (1991).

The last two columns of Table 2 report the results of estimating the same MA(1)-GARCH(1,1) model in (3.2) for each of the one thousand realizations of the 720 monthly simulated Null2 and TVRP returns. These are the only two models that exhibit significant heteroskedasticity as evidenced by the $Q_{12}^{(2)}$ statistic in Table 1. The similarity between the estimated GARCH coefficients for the artificially generated returns and the actual data is striking. The median values of the quasi-maximum-likelihood estimates of α over the

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[19] In the EGARCH(1,1) model the conditional variance is given by

$$\ln(\sigma_t^2) = \omega + \theta z_{t-1} + \gamma(|z_{t-1}| - E(|z_{t-1}|)) + \beta \ln(\sigma_{t-1}^2),$$

where $z_t \equiv \varepsilon_t\sigma_t^{-1}$ denotes the standardized innovations. Alternative asymmetric conditional variance formulations include the model in Glosten, Jagannathan, and Runkle (1990) and the specifications in Engle and Ng (1991). Recent evidence in Gallant, Rossi, and Tauchen (1992) and Andersen (1992) explores structural links between conditional volatility and volume. Space considerations prevent us from pursuing these specifications here.

one thousand replications for the Null2 and TVRP alternatives are 0.108 and 0.127, respectively, compared to 0.123 for the real data. Similarly, the median values for the estimates of β are 0.865 and 0.853, respectively, compared to 0.861 for the actual returns. Note also that although the Null2 alternative is unable to explain the negative skewness in the standardized returns, the TVRP hypothesis results in excess negative skewness and excess leptokurtosis compared to the real data.

In summary, the results in Table 2 illustrate how explicitly allowing for time-varying uncertainty in the fundamental real dividend growth process within the context of a simple present value relationship may endogenously account for the observed ARCH effects in the data. Reconciliation of a fundamental model and the volatility of short-horizon real returns appears to require time variation in the discount factor.

4. Long-Horizon Tests

The previous section documents that the evidence in autocorrelations of short-horizon returns against the hypothesis of a constant conditional mean return is not very strong. At the same time, the results indicate that returns appear to be too volatile relative to the models with a constant discount factor. Shiller (1984) and Summers (1986) argued that Fads would make returns more volatile. However, it would be difficult to detect a Fads alternative hypothesis, as developed above, with autocorrelation tests because of their low power when transitory components are very highly serially correlated. Poterba and Summers (1988) and Fama and French (1988a, 1988b) also realized that the negative serial correlation in returns implied by such a model would manifest itself more transparently at longer horizons. Consequently, Poterba and

Summers (1988) investigated long-horizon variance ratios as in equation (3.1), while Fama and French (1988a) analyzed regressions of long-horizon returns on lagged long-horizon returns.

The statistical properties of the Fama and French (1988a) analysis have generated much controversy in the literature. For example, Jegadeesh (1990), Kim, Nelson, and Startz (1989), Mankiw, Romer, and Shapiro (1991), Richardson (1990), and Richardson and Stock (1989) all argue that the case for predictability of long-horizon stock returns is weak when one corrects for the small sample biases in the test statistics. Our simulations demonstrate these biases. We then present two additional ways that hypotheses regarding the long-horizon predictability of returns can be investigated which are not as severely biased. These techniques apply generally in other long-horizon forecasting situations.

To begin, let $\ln(R_{t+k,k}) \equiv \ln(R_{t+1}) + \cdots + \ln(R_{t+k})$ denote the continuously compounded k-period rate of return. Then, a typical OLS specification of Fama and French (1988b) is the following:

$$\ln(R_{t+k,k}) = \alpha_{k,k} + \beta_{k,k}\ln(R_{t,k}) + u_{t+k,k}. \tag{4.1}$$

Note that the k-period error term $u_{t+k,k}$ is not realized until time t+k. Therefore, if the data are sampled more finely than the compound return interval, $u_{t,t+k}$ is serially correlated, even under the null hypothesis of constant expected returns. If all of the data are employed, $u_{t+k,k}$ is correlated with k-1 previous error terms as discussed in Hansen and Hodrick (1980). If the one-period returns are serially uncorrelated, it is possible to solve explicitly for the parameters in the corresponding moving average representation for $u_{t+k,k}$ as a function of the overlap as in Baillie and Bollerslev (1990). Under alternative hypotheses in which returns have a variable conditional mean, however, $u_{t+k,k}$ can be arbitrarily serially correlated if lagged returns do not capture all of the variation in the conditional mean.

Since lagged returns are predetermined but not strictly exogenous, asymptotic distribution theory must be used to determine the properties of an estimator for $\kappa_{k,k} = (\alpha_{k,k}, \beta_{k,k})'$. Ordinary least squares provides consistent estimates, but traditional OLS standard errors are not appropriate asymptotically since the error term is serially correlated when forecasting more than one period ahead. Furthermore, the variability of the conditional variance of returns, documented in Section 3, makes it inappropriate to assume homoskedasticity which underlies the derivation of the conventional OLS standard errors.

Nevertheless, the OLS coefficient estimator from equation (4.1), $\hat{\kappa}_{k,k}$, is readily interpreted as a generalized method of moments (GMM) estimator based on the instruments $x_t = (1, R_{t,k})'$. Hence, it is straightforward to derive appropriate asymptotic standard errors. Following Hansen (1982), it can be demonstrated that $\sqrt{T}(\hat{\kappa}_{k,k} - \kappa_{k,k}) \sim N(0, \Omega)$, where $\Omega = Z_0^{-1} S_0 Z_0^{-1}$, $Z_0 = E(x_t x_t')$, and S_0 denotes the spectral density evaluated at frequency zero of $w_{t+k,k} = u_{t+k,k} x_t$. Under the null hypothesis that the returns are not predictable,

$$S_0 = \sum_{j=-k+1}^{k-1} E(w_{t+k,k} w'_{t+k-j,k}). \tag{4.2}$$

This matrix is consistently estimated by

$$S_T^* = C_T(0) + \sum_{j=1}^{k-1} [C_T(j) + C_T(j)'], \tag{4.3}$$

where $C_T(j) = (1/T)\Sigma_{t=j+1}^{T}(\hat{w}_{t+k,k}\hat{w}'_{t+k-j,k})$, and $\hat{w}_{t,k}$ denotes $w_{t,k}$ evaluated at the estimated residuals. Similarly, a consistent estimator for Z_0 is given by $Z_T = (1/T)\Sigma_{t=1}^{T} x_t x_t'$.

We present the results for this estimation of equation (4.1) for five horizons in Panel A of Table 3. The first column indicates the horizon k equal to 1, 12, 24, 36, and 48 months. As in the previous tables, the second column labelled Data provides the sample statistics, which in this case are the OLS estimates of the slope coefficients, $\beta_{k,k}$, with asymptotic standard error in parentheses. The p-values of the t-tests for the hypothesis $\beta_{k,k} = 0$ from the asymptotic distribution and the five simulated economies are presented in the next six columns. The median values of the test statistics are in square brackets.

Notice that at the twenty-four month horizon, the asymptotic p-value is effectively zero, while the one-month and thirty-six month values both equal 0.162. Richardson (1990) notes that considerable care must be exercised when examining the individual test statistics since they are highly correlated. Hence, we also report the $\chi^2(5)$ statistic that examines the joint test that all five slope coefficients are zero. It has a value of 15.8, which corresponds to an asymptotic p-value of 0.007. Clearly, if the asymptotic distributions are validated by the simulations, this would be strong evidence of long-horizon return predictability.

Examination of the columns labelled Null1 and Null2 indicates that the asymptotic distribution is a poor approximation to the distribution of the test statistics in small samples. The median values of the sample $\chi^2(5)$ statistics for these two simulated economies are 8.61 and 11.1, respectively, compared to the median of a true $\chi^2(5)$ of 4.35. The corresponding p-values from the two distributions are 0.237 and 0.324. Hence, there is actually little evidence against the null hypothesis. The deterioration of the t-tests is apparent as the horizon is increased, with the median values becoming increasingly negative.[20]

Notice also that the p-value and the median value of the $\chi^2(5)$ statistics for the Bubble model are close to those of the Null1 and Null2 simulations. This is not surprising, as the Bubble model maintains a constant expected return. More surprisingly, perhaps, are the p-values for the Fads alternative which are only slightly larger than those under the Null1 hypothesis. This latter finding illustrates the low power of these tests when Type I error rates are fixed at the traditional levels. Given the large overlap in the distributions

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[20] Richardson and Stock (1989) develop an alternative asymptotic distribution theory based on a functional central limit theorem for long-horizon return regressions. They argue that if T is the sample size and k is the forecast horizon, a more appropriate asymptotic distribution is derived by letting (k/T) go to a non-zero constant rather than to zero as in the conventional asymptotic distribution theory. Inference under this alternative distribution theory provides little support for the hypothesis that lagged long-horizon returns predict future long-horizon returns. Nelson and Kim (1991) and Hodrick (1992) use Monte Carlo simulations and Goetzmann and Jorion (1992) use bootstrap techniques to investigate the regressions of dividend yields as predictors of long-horizon returns. These authors find that the small sample properties of estimators are not well approximated by conventional asymptotic distribution theory.

of the test statistics under the null and the alternative hypotheses, the probability of failing to reject the null hypothesis when it is actually false is quite high. Finally, notice that the p-value for the $\chi^2(5)$ statistic for the TVRP model is 0.460 which is the largest of any of the models.

The bottom part of Panel A reports the coefficients of multiple correlation, R^2 statistics, for the five horizons. Although the sample values increase from 0.009 at the one-month horizon to 0.058 at the thirty-six month horizon, the smallest p-values of the simulated economies are actually at the one-month horizon. The serial correlation of the residuals induced by using overlapping forecasting intervals causes a spurious regression phenomenon as in Granger and Newbold (1974).

We next develop an alternative estimator of S_0 that is valid only under the null hypothesis. This estimator utilizes the fact that the values of unconditional expectations of covariance stationary time series depend only on the time intervals between the observations.[21] In particular, notice that under the null hypothesis, $u_{t+k,k} = (e_{t+1} + \cdots + e_{t+k})$, where e_{t+1} is the serially uncorrelated one-step-ahead forecast error of returns. Estimates of e_{t+1} can be obtained from the residuals of a regression of $\ln(R_{t+1})$ on a constant because under the null hypothesis $e_{t+1} = u_{t+1,1}$. To derive the alternative estimator, examine a typical term in equation (4.2), $E(w_{t+k,k}w'_{t+k-j,k})$, where $k > j > 0$. Substituting $(e_{t+1} + \cdots + e_{t+k})$ for $u_{t+k,k}$,

[21] Lars Hansen suggested this estimator, which is a heteroskedastic counterpart to the covariance matrix in Richardson and Smith (1989). This section draws heavily from Hodrick (1992).

$$E(u_{t+k,k}x_t u_{t+k-j,k}x'_{t-j}) = E[(\sum_{i=1}^{k} e_{t+i})x_t(\sum_{h=1-j}^{k-j} e_{t+h})x'_{t-j}]$$
$$= E[(\sum_{i=1}^{k-j} e_{t+i}^2)x_t x'_{t-j}]. \tag{4.4}$$

With stationary time series, the unconditional expectation of each of the (k - j) terms on the right-hand side of equation (4.4) depends only upon the time interval between the variables. Hence, rather than summing e_{t+i} into the future, one can sum $x_t x'_{t-j}$ into the past:

$$E[(\sum_{i=1}^{k-j} e_{t+i}^2)x_t x'_{t-j}] = E[e_{t+1}^2(\sum_{i=0}^{k-j-1} x_{t-i}x'_{t-j-i})]. \tag{4.5}$$

Applying the same logic to all of the terms in equation (4.2) implies that

$$S_0 = E[e_{t+1}^2(\sum_{i=0}^{k-1} x_{t-i})(\sum_{i=0}^{k-1} x_{t-i})'] = E(v_{t+1,k}v'_{t+1,k}) \tag{4.6}$$

where

$$v_{t+1,k} \equiv e_{t+1}(\sum_{i=0}^{k-1} x_{t-i}). \tag{4.7}$$

Let $\hat{v}_{t+1,k}$ denote $v_{t+1,k}$ evaluated at the estimated residual, $\hat{e}_{t+1}$. An alternative estimator for S_0 from equation (4.6) is then,

$$S_T^b = \frac{1}{T}\sum_{t=k}^{T} \hat{v}_{t+1,k}\hat{v}'_{t+1,k}. \tag{4.8}$$

Two aspects of the estimator S_T^b are important, and both are induced by the fact that it avoids the summation of autocovariance matrices as in equation (4.3). First, the estimator is guaranteed to be positive definite. Second, if it is the summation of the autocovariance matrices that causes the poor small sample properties of the test statistics in Panel A of Table 3, the finite sample behavior of test statistics constructed with S_T^b might be better.

The properties of this estimator are investigated in Panel B of Table 3. The point estimates of the slope coefficients reported in the Data column are identical, but the standard errors are different from Panel A. In particular, the test statistic at the twenty-four month horizon now has an asymptotic p-value of 0.411 and the $\chi^2(5)$ statistic is only 3.37, below the median value of 4.35 in the asymptotic distribution. However, 3.37 is actually larger than the median values of the small sample distributions for the test statistics from the Null1 and Null2 simulations. Also, notice that the deterioration of the median values of the test statistics is mitigated only slightly.

The last part of this section demonstrates how inference about the statistical significance of lagged returns as predictors of long-horizon returns can be conducted by considering the regression of one-period returns on the weighted sum of the lagged returns.[22] This specification also avoids the summation of autocovariance matrices and may therefore have better small sample properties under the null hypothesis than the estimates based on equation (4.1).

Since the compound k-period return is the sum of k one-period returns, the numerator of the regression coefficient $\beta_{k,k}$ in equation (4.1) is an estimate of $\mathrm{cov}[\Sigma_{j=1}^{k}\ln(R_{t+j}); \Sigma_{j=0}^{k-1}\ln(R_{t-j})]$. This covariance is the weighted sum of (2k - 1) autocovariances of returns separated by between one and (2k - 1) periods. With covariance stationary time series it follows that

$$\mathrm{cov}[\sum_{j=1}^{k}\ln(R_{t+j}); \sum_{j=0}^{k-1}\ln(R_{t-j})] = \mathrm{cov}[\ln(R_{t+1}); \sum_{j=1}^{2k-1}\omega_j \ln(R_{t+1-j})], \tag{4.9}$$

where $\omega_j = j$ for $1 \leq j \leq k$, and $\omega_j = 2k - j$ for $(k + 1) \leq j \leq (2k - 1)$. The sum of the covariances on the right-hand side of equation (4.9) is equal to the numerator of the slope coefficient in the following regression:

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[22] Jegadeesh (1990) uses this logic and the Fads alternative hypothesis to derive the test with the largest asymptotic slope for investigating long-horizon predictability of returns on the basis of lagged returns. He argues that using the one-period return as the dependent variable and the sum of k lagged returns as the regressor is a superior way to conduct inference. The choice of k depends on the share of the variance of returns attributed to the transitory components in prices.

$$\ln(R_{t+1}) = \alpha_{1,2k-1} + \beta_{1,2k-1}\left[\sum_{j=1}^{2k-1} \omega_j \ln(R_{t+1-j})\right] + u_{t+1}. \qquad (4.10)$$

Under the null hypothesis, the error term in equation (4.10) is serially uncorrelated in contrast to $u_{t+k,k}$ in equation (4.1). The asymptotic distribution of the OLS estimator for $\kappa_{1,h} = (\alpha_{1,h}, \beta_{1,h})'$ can be derived as above. Since only the term corresponding to j = 0 is different from zero in equation (4.2), this specification might have better small sample properties under the null hypothesis.

The results of estimating $\beta_{1,2k-1}$ in equation (4.10) are presented in Table 4. The small sample properties of the test statistics are better than the results of the tests in Panel A of Table 3 and are comparable to those in Panel B of Table 3. The deterioration in the medians of the individual test statistics is again quite evident, even though the p-values for the asymptotic distribution and for the Null1 and Null2 models are quite close. There is little evidence for predictability of returns. Again, notice from the closeness of the medians for the Fads model and the TVRP model that these tests are likely to have very low power. We note here that the results in this section only provide evidence regarding the own-predictability of returns and do not demonstrate that returns are unpredictable when additional information is used. We return to this issue in Section 6 below.

5. Cointegration Tests

Recently, a number of authors have proposed various tests for the presence of speculative bubbles and market efficiency that are based on the idea of cointegration as discussed in Engle and Granger (1987). In light of space constraints and related coverage elsewhere in this volume, our discussion of these techniques is brief.[23] Intuitively, two time series are defined to be cointegrated if each of the individual series is non-stationary, yet a linear combination of the two series is stationary.

From the present value model in equation (2.1),

$$\frac{P_t^f}{D_t} = E_t\left(\rho_{t+1}\left(\frac{P_{t+1}^f}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\right). \tag{5.1}$$

Assuming the transversality condition is satisfied, and using $D_{t+1}/D_t = 1 + \Delta d_{t+1}$, it follows that

$$\frac{P_t^f}{D_t} = E_t\left(\sum_{j=1}^{\infty}\prod_{i=1}^{j}\rho_{t+i}(1 + \Delta d_{t+i})\right). \tag{5.2}$$

If the dividend growth rate, Δd_t, and the discount factor, ρ_t, are jointly covariance stationary, $p_t^f \equiv \log(P_t^f)$ and d_t will be cointegrated with cointegrating vector (1, -1); see Cochrane (1991a) for a formal proof. If the actual market price contains a rational speculative bubble as in equation (2.13), $P_t = P_t^f + B_t$. The expectation of an explosive bubble would violate the transversality condition, and $p_t - d_t$ would be non-stationary. It is worth noting that this argument does not depend on any particular equilibrium fair rate of return but

[23] See Campbell and Shiller (1987, 1988a, 1988b), Cochrane (1991a, 1992), Craine (1991), Evans (1991), and Froot and Obstfeld (1992) for more complete discussions and applications.

only requires that the implied discount factor and the dividend growth rate are jointly covariance stationary.

The augmented Dickey-Fuller t-tests reported in Panel A of Table 5, confirm that the logarithms of real prices and real dividends are non-stationary or integrated of order one. From Fuller (1976) the one and five percent critical values in the corresponding asymptotic unit root distribution are -3.43 and -2.86, respectively. Hence, the corresponding test statistics of -1.83 and -1.35 for the logarithms of real prices and real dividends cannot reject the null hypothesis of a unit root in the autoregressive polynomial in the univariate time series representation for either of the two time series. The presence of a unit root also underlies modeling the dividend growth rate as a stationary process in equation (2.5). The conformity among the different p-values in the table illustrates the robustness of the standard unit root tests to the presence of conditional heteroskedasticity as shown by Phillips (1987).

The tests for cointegration of p_t and d_t are presented in Panel B of Table 5. Following Engle and Granger (1987), if p_t and d_t are not cointegrated, the residuals from the cointegrating regression of p_t on a constant and d_t will contain a unit root. Note that under the null hypothesis of no cointegration the regression of p_t on d_t is spurious in the sense of Granger and Newbold (1974). The asymptotic distribution of this residual based unit-root test has been formally derived by Phillips and Ouliaris (1990).[24] The one and five percent critical values are -3.96 and -3.37, respectively. Consistent with

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[24] The likelihood ratio test in Johansen (1988) provides an alternative non-residual-based testing procedure for cointegration.

the predictions of the present value model and the absence of speculative bubbles, the null hypothesis of a unit root in the mean-zero residuals from the cointegrating regression is easily rejected since the test statistic is -4.57.

When we impose the cointegrating vector (1, -1), the null hypothesis of a unit root in the logarithmic price-dividend ratio is also firmly rejected. Note that, when imposing the cointegrating vector, the corresponding test statistic should be evaluated in the standard asymptotic unit-root distribution with one and five percent critical values of -3.43 and -2.86. A time series plot of the logarithmic price-dividend ratio is given in Figure 3.

These results are counter to related findings reported in the literature, which typically fail to reject the null of no cointegration using annual data. The findings in Table 5 for monthly data suggest that these results may be due to a lack of power in the tests using time-aggregated annual data, even though the annual span of the data is comparable. Note that while the p-values for the Null1 model suggest much more powerful rejections, the p-value for the TVRP model is broadly consistent with the actual empirical findings. The residual-based cointegration test statistic for the actual data is also much lower than the values obtained for the Fads and Bubble models. At the same time, the median values of the test statistic from the empirical distributions of the Bubble model suggest that the cointegration test is not very powerful against the collapsing bubble analyzed here. Additional evidence on this issue is provided in Evans (1991). Similarly, based on the results for the TVRP model, the test for no cointegration is likely to have low power in the presence of a time-varying discount rate.

Even though the estimate of b from the cointegrating regression is significantly different from the implied value of unity in the TVRP model at the 0.024 level, it is interesting to note that all of the other alternatives, including the Bubble specification, result in even lower p-values for the estimated coefficient $\hat{b} = 1.31$.[25] It follows also that $\hat{a} = 3.94$ is too small compared to the results for the five simulated price processes. From equation (2.1), if real monthly dividends followed a logarithmic random walk with drift μ and normal innovations with a variance of σ^2 and the required rate of return were the constant r,

$$p_t^f - d_t = -\ln[\exp(r - \mu - 0.5\sigma^2) - 1]. \tag{5.3}$$

When evaluated at the sample analogues of $\mu = 0.00086$, $\sigma^2 = 0.000311$, and $r = 0.00635$, the right-hand side of equation (5.3) equals 5.23. The median values for the intercept under each of the alternatives are all close to this value.

6. Volatility Tests

In the late 1970's researchers interested in the efficiency of asset markets shifted their focus from the predictability of returns to the volatility of prices. As always, the hypothesis of market efficiency could not be tested directly but was part of a joint hypothesis. Researchers were still required to specify a particular model of expected returns. Additionally, the predictions

[25] As noted in footnote 2, if the importance of dividends as a means of distributing cash to shareholders has declined systematically, one would expect that the slope coefficient in the cointegrating regression would exceed one.

of price volatility from a particular model depended on the assumed time series properties of the dividend process and the information set of economic agents.

The first volatility tests were conducted by Shiller (1979, 1981) and LeRoy and Porter (1981). These authors assumed a constant expected rate of return model and reported overwhelming rejections of market efficiency. Subsequent research, particularly by Flavin (1983) and Kleidon (1986a,b), questioned the small sample statistical properties of these analyses. For recent surveys of this literature see West (1988), Pesaran (1991), Gilles and LeRoy (1991) and Cochrane (1991a).

In order to illustrate the issues, we focus here on a volatility test developed by Mankiw, Romer and Shapiro (1991), which we denote the MRS test. This class of tests is designed to avoid biases that plagued previous studies and to provide statistically reliable standard errors for the test statistics. Like many volatility tests, the MRS test recognizes that in an efficient market the price of an asset must equal the discounted conditional expected payoff from holding the asset for k periods and reselling it. That is,

$$P_t = E_t(P_t^{*k}) \tag{6.1}$$

where,

$$P_t^{*k} \equiv \sum_{i=1}^{k} \rho_{t+1}^{i} D_{t+i} + \rho_{t+i}^{k} P_{t+k}. \tag{6.2}$$

Note that the ex-post rational price defined in equation (6.2) is only observable at time t + k.[26]

Equation (6.1) implies that P_t^{*k} - P_t is uncorrelated with any information available at time t. In particular, let P_t^0 denote any "naive forecast" of the ex-post rational price. Then, the following second-order moment condition must hold:

$$E_t[(P_t^{*k} - P_t)(P_t - P_t^0)] = 0. \tag{6.3}$$

This in turn implies that

$$E_t[(P_t^{*k} - P_t^0)^2] = E_t[(P_t^{*k} - P_t)^2] + E_t[(P_t - P_t^0)^2]. \tag{6.4}$$

Equation (6.4) remains valid when the price constructs are deflated by any variable in the time t information set. As the results in Table 5 indicate, such a transformation is necessary to ensure stationarity and the existence of unconditional expectations required in deriving the test statistics. In our implementation of the MRS volatility test we follow their lead and divide by P_t. Hence, from equation (6.4),

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[26] This contrasts with Shiller's (1981) definition of ex post rational price in which $k = \infty$ in equation (6.1). Shiller develops a measurable counterpart by substitution of k_t = T - t for k in equation (6.2) where T is the end of the sample observed by the econometrician. Flood and Hodrick (1990) refer to equation (6.1) as an iterated Euler equation.

$$E_t\left[\left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2\right] - E_t\left[\left(\frac{P_t^{*k}}{P_t} - 1\right)^2\right] - E_t\left[\left(1 - \frac{P_t^0}{P_t}\right)^2\right] = 0. \tag{6.5}$$

Let q_{t+k} denote the corresponding sample realizations of equation (6.5):

$$q_{t+k} \equiv \left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2 - \left(\frac{P_t^{*k}}{P_t} - 1\right)^2 - \left(1 - \frac{P_t^0}{P_t}\right)^2. \tag{6.6}$$

The null hypothesis from equation (6.5) is then stated as $E_t(q_{t+k}) = 0$.

By the law of iterated expectations, $E(q_t) = 0$, and the sample mean of q_t, denoted $\bar{q}$, should be close to zero under the null hypothesis. The asymptotic standard error for $\bar{q}$ may be constructed, as in Section 4, by use of the GMM distribution theory for stationary processes that are serially correlated and conditionally heteroskedastic. That is,

$$\sqrt{T}\,\bar{q} \sim N(0, V_0), \tag{6.7}$$

where

$$V_0 = \sum_{j=-k+1}^{k-1} E(q_{t+k}q_{t+k-j}). \tag{6.8}$$

The sample variance may be estimated by

$$V_T = C_T(0) + \sum_{j=1}^{k-1} 2C_T(j), \tag{6.9}$$

where $C_T(j) = (1/T)\Sigma_{t=j+1}^{T}(q_{t+k}q'_{t+k-j})$. In order to guarantee a positive estimate of the variance in equation (6.9), the j-th autocovariance is weighted by (k-j)/k as in Newey and West (1987).

As our measure of a naive price prediction, we use a version of the Gordon (1962) model assuming a constant rate of return, r, and a constant dividend growth rate, μ,

$$P_t^0 = \frac{D_t}{r - \mu}. \tag{6.10}$$

The tests are reported in Panel A of Table 6 for the same five horizons as in Table 4. Notice in the Data column that $\bar{q}$ is negative at all five horizons.[27] The asymptotic p-values are also quite small with the largest being 0.023 at the 24 month horizon. Given the strong significance for each of the individual MRS test statistics at all horizons, we did not calculate a joint test.

Several features of the MRS test are noteworthy in the simulated data. First, for both the Null1 and Null2 models, there is no bias in the test statistics in the sense that the median values of the test statistics are essentially zero. This desirable feature of the MRS volatility test arises because of its use of

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[27] We use the sample mean return for r and the sample mean of the dividend growth rate for μ, rather than preselected values, in the construction of the naive prices.

uncentered second moments rather than sample variances which are biased because of the necessity of estimating the sample mean. Next, notice that the p-values for the Null1, Null2, Fads, and Bubble models are all quite small. It is unlikely that any of these models could generate the sample volatility statistics. Only for the TVRP model are the p-values reasonable. Here we find values ranging from 0.142 to 0.346.

The orthogonality condition in equation (6.3) also forms the basis for Scott's (1985) regression test of the present value model. If equation (6.1) is true, a regression of $(P_t^{*k} - P_t)$ on anything in the time t information set should have insignificant coefficients. Note again, that deflation of the price series by some W_t in the time t information set does not formally change this null hypothesis but is desirable to ensure stationarity. In particular, consider the regression specified without a constant term as

$$\frac{P_t^{*k} - P_t}{W_t} = \beta\left(\frac{P_t - P_t^0}{W_t}\right) + e_t. \tag{6.11}$$

The OLS estimate for β is

$$\beta = \frac{\frac{1}{T}\sum_{t=1}^{T}\left[(P_t^{*k} - P_t)/W_t\right]\left[(P_t - P_t^0)/W_t\right]}{\frac{1}{T}\sum_{t=1}^{T}\left[(P_t - P_t^0)/W_t\right]^2}. \tag{6.12}$$

For $W_t = P_t$ the numerator of β equals $(1/2)\bar{q}$. Mankiw, Romer and Shapiro (1991) note that their volatility test has an advantage and a disadvantage relative to the regression test. The disadvantage is that regression coefficients and the coefficient of multiple correlation, R^2, have natural interpretations in terms of the predictability of returns. The advantage is that regression tests may be more systematically biased.

Equation (6.5) implies two volatility inequalities which serve as diagnostics for the models:

$$E\left[\left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2\right] \geq E\left[\left(\frac{P_t^{*k}}{P_t} - 1\right)^2\right], \tag{6.13}$$

and

$$E\left[\left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2\right] \geq E\left[\left(1 - \frac{P_t^0}{P_t}\right)^2\right]. \tag{6.14}$$

Panel B of Table 6 reports the sample analogue for the left-hand side of equations (6.13) and (6.14) as ex-post rational price relative to naive price. The realization of the right-hand side of equation (6.13) is denoted ex-post rational price relative to unity, while the sample realization of the right-hand side of equation (6.14) is referred to as naive price relative to unity. Interestingly, it is the latter quantity that is the primary source of evidence for excess volatility in the data. From inspection of the p-values for the various models, it follows that only the TVRP model is consistent with the data at a ten percent significance level.

The right-hand side of equation (6.14) is a measure of the variability in the dividend-price ratio. The naive Gordon (1962) model predicts that the dividend-price ratio should be a constant. There are three sources of movement in the TVRP model that improve on this counterfactual prediction. First, information other than the current level of dividends is useful for predicting future dividends, which is true for all the models. Second, the time variation in the conditional variance of dividends matters to investors who are forecasting the levels of dividends, which is true for Null2 as well. The third feature is that the required rate of return is not a constant.[28]

As noted above, there is also a direct relation between the volatility of prices and the predictability of returns. To understand this relation consider the following argument. Assuming a constant discount factor ρ, and $W_t = P_t$, it follows from equation (6.2) that for $k = 2$,

$$\frac{P_t^{*2}}{P_t} - 1 = \rho\frac{D_{t+1}}{P_t} - 1 + \rho^2\left(\frac{D_{t+2} + P_{t+2}}{P_t}\right). \tag{6.15}$$

Add and subtract $\rho(P_{t+1}/P_t)$ to the right-hand side of equation (6.15) and multiply the second term on the right-hand side by (P_{t+1}/P_{t+1}). The result is

[28] Several authors including Campbell (1991), Cochrane (1992), and LeRoy and Parke (1992) have reformulated volatility tests to examine the variance of the price-dividend ratio.

$$\frac{P_t^{*2}}{P_t}-1 = \rho\left(\frac{D_{t+1}+P_{t+1}}{P_t}\right)-1+\rho^2\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{D_{t+2}+P_{t+2}}{P_{t+1}}\right)-\rho\frac{P_{t+1}}{P_t}$$
$$= (\rho R_{t+1}-1)+\rho\left(\frac{P_{t+1}}{P_t}\right)(\rho R_{t+2}-1). \tag{6.16}$$

Clearly, if the null hypothesis is true and expected returns have a constant mean equal to ρ^{-1}, $(P_t^{*2}/P_t - 1)$ is not predictable.

Let $x_t \equiv (1 - P_t^0/P_t)$, and let $u_{t+1} \equiv (\rho R_{t+1} - 1)$. Substituting these definitions into equation (6.16) and the results into equation (6.3) with its variables deflated by P_t yields

$$E_t[(u_{t+1} + \rho(P_{t+1}/P_t)u_{t+2})x_t] = 0. \tag{6.17}$$

It is apparent that both the MRS volatility test and Scott's regression test examine the null hypothesis that a variable in the time t information set cannot predict returns at various horizons.

As in Section 4, stationarity of the variables in equation (6.17) allows us to reorganize the equation and express its unconditional expectation as

$$E[u_{t+1}(x_t + \rho(P_t/P_{t-1})x_{t-1})] = 0. \tag{6.18}$$

This formulation of the volatility test makes it transparent that it is predictability of one period returns that is being tested, and from the definition of x_t, any predictability of returns is due to a filtered measure of dividend yields. The sample counterparts to the unconditional expectation of equation (6.17) and equation (6.18) only differ by the first and last observations. Of course, estimates of the variance of the sample mean might be very different in small samples. The variance for the estimator based on equation (6.17) may be calculated as in equation (6.9). However, the variance for the estimator based on equation (6.18) avoids the overlapping data problem and the summation of the autocovariances and may be calculated analogously to equation (4.8). The small sample properties of resulting test statistics will therefore differ. We conjecture that reorganizing the test statistics to avoid the overlapping data problem will result in superior small sample behavior.

7. Conclusion

The analysis in this chapter provides a partial survey of the econometric methods employed in testing for own temporal dependencies in the distribution of asset returns. It also addresses the relationship of these dependencies to the concept of market efficiency. We intentionally worked only with price, dividend, and return data, and avoided the use of other macroeconomic variables that might help explain the evolution of returns over time. We focussed the discussion on tests that have been primarily design for broadly defined asset categories. A large literature in finance analyzes cross-sectional differences in returns. Fama and French (1992) and Ferson (1992) provide recent contributions to this literature. Our somewhat narrow focus was motivated in part by space considerations and by the relevance of expected return variability for issues in macroeconomics.

Much of the literature on testing for market efficiency has proceeded under the convenient assumption that rational asset pricing requires a constant rate of return. While simple short-horizon serial correlation tests often cannot reject this hypothesis, time-variation in equilibrium required rates of returns has been predicted by rational general equilibrium theories since the early models of LeRoy (1973), Lucas (1978), and Breeden (1979). These models are often referred to as consumption based capital asset pricing models.

Although the time-varying risk premium model postulated in our simulations was not rigorously derived from a rational expectations model, its performance in the simulations was broadly consistent with the empirical findings pertaining to U.S. stock returns. In contrast, the alternative models assuming a constant discount factor or modifications to incorporate fads or bubbles in asset prices were grossly inconsistent with some aspects of the data. These results illustrate the importance of explicitly recognizing the presence of a time-varying risk premium in tests for market efficiency.

The particular functional form for our time-varying risk premium model related the expected return on the market to the change in the conditional variance of future dividends. This formulation was motivated by the analysis in Abel (1988) and Hodrick (1989). Such an approach is radically different from much of the empirical literature that has been devoted to testing restrictions implied by the consumption based capital asset pricing model. In spite of its theoretical appeal, the consumption based CAPM does not perform well empirically, as exemplified by the many tests following the approach of Hansen and Singleton (1982, 1983). The basic problem stems from the fact that consumption is much too smooth compared to the variability of most financial asset returns. This is formally documented in Hansen and Jagannathan (1991) who derive bounds on intertemporal marginal rates of substitution using asset market data. In a related context, Pesaran and Potter (1991) argue that predictability of negative excess returns is inconsistent with most equilibrium consumption-based asset pricing models. While the consumption-based asset pricing models generally fail specification tests, the more recent developments in Cochrane (1991b) and Braun (1991) suggest that alternative dynamic asset pricing models based on the investment decisions of firms may provide a better explanation for the observed time-varying risk premia.

The ready availability of data on asset returns and the declining cost of computing have generated a large literature documenting the stylized facts of financial markets. The current challenge facing financial economists is to develop models that are consistent with the observed variability in the distributions of returns. Variability of expected returns appears to be a necessary aspect of rational explanations of these phenomena. We also conjecture that understanding the link between conditional volatility of market fundamentals and variation in required expected returns that arises from risk aversion of economic agents will be critical to success in this endeavor.

Appendix

Forecast expressions for $E_t(d_{t+j})$ and $Var_t(d_{t+j})$ from the estimated AR(12)-ARCH(12) model for the dividend growth rate in equation (2.5) are most easily evaluated by expressing the model in first-order companion form in the logarithmic levels. That is,

$$\begin{bmatrix} d_t \\ d_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ d_{t-12} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1+1 & \phi_2-\phi_1 & \cdots & \phi_{12}-\phi_{11} & -\phi_{12} \\ 1 & 0 & \cdots & 0 & 0 \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} d_{t-1} \\ d_{t-2} \\ \cdot \\ \cdot \\ \cdot \\ d_{t-13} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

or more compactly,

$$\underline{d}_t = \underline{\mu} + \underline{\Phi}\,\underline{d}_{t-1} + \underline{\varepsilon}_t, \tag{A1}$$

where μ denotes the constant, and ϕ_i the i-th autoregressive parameter for Δd_t in equation (2.5).

By repeated substitution in equation (A1),

$$\underline{d}_{t+j} = \sum_{i=0}^{j-1}\underline{\Phi}^i\,\underline{\mu} + \sum_{i=0}^{j-1}\underline{\Phi}^i\,\underline{\varepsilon}_{t+j-i} + \underline{\Phi}^j\,\underline{d}_t.$$

Define $\underline{e}_i$ to be a 13×1 basis vector of zeros except for unity in the i-th element, and $\psi_i \equiv \underline{e}_1'\,\underline{\Phi}^i\,\underline{e}_1$. Then,

$$\begin{aligned} d_{t+j} &= \mu\sum_{i=0}^{j-1}\underline{e}_1'\,\underline{\Phi}^i\,\underline{e}_1 + \sum_{i=0}^{j-1}\underline{e}_1'\,\underline{\Phi}^i\,\underline{e}_1\varepsilon_{t+j-i} + \underline{e}_1'\,\underline{\Phi}^j\sum_{i=1}^{13}\underline{e}_i\,d_{t+1-i} \\ &= \mu\sum_{i=0}^{j-1}\psi_i + \sum_{i=0}^{j-1}\psi_i\varepsilon_{t+j-i} + \underline{e}_1'\,\underline{\Phi}^j\sum_{i=1}^{13}\underline{e}_i\,d_{t+1-i}. \end{aligned} \tag{A2}$$

It follows now directly from equation (A2) that

$$E_t(d_{t+j}) = \mu \sum_{i=0}^{j-1} \psi_i + \underline{e}_1{}' \Phi^j \sum_{i=1}^{13} \underline{e}_i \, d_{t+1-i}, \tag{A3}$$

and

$$\mathrm{Var}_t\,(d_{t+j}) = \sum_{i=0}^{j-1} \psi_i^2 \, E_t(\varepsilon_{t+j-i}^2). \tag{A4}$$

Evaluation of the expression for the conditional variance of d_{t+j} requires forecasts from the ARCH(12) conditional variance process for ε_t. Following Baillie and Bollerslev (1992), the ARCH(12) model is conveniently expressed in first-order companion form as,

$$\begin{bmatrix} \varepsilon_t^2 \\ \varepsilon_{t-1}^2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{t-11}^2 \end{bmatrix} = \begin{bmatrix} \omega \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{12} \\ 1 & 0 & \cdots & 0 \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1}^2 \\ \varepsilon_{t-2}^2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{t-12}^2 \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix},$$

or compactly as,

$$\underline{\varepsilon}_t^2 = \underline{\omega} + \underline{\Gamma}\, \underline{\varepsilon}_{t-1}^2 + \underline{\nu}_t, \tag{A5}$$

where $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$. Note, $E_{t-1}(\nu_t) = 0$, and ν_t is readily interpreted as the innovation to the conditional variance for ε_t. Analogous to the expression for d_{t+j} in equation (A2), it follows by repeated substitution in equation (A5) and post-multiplication with the 12×1 basis vector $\underline{e}_1$, that

$$\varepsilon_{t+j}^2 = \omega \sum_{i=0}^{j-1} \underline{e}_1{}' \underline{\Gamma}^i \underline{e}_1 + \sum_{i=0}^{j-1} \underline{e}_1{}' \underline{\Gamma}^i \underline{e}_1 \, \nu_{t+j-i} + \underline{e}_1{}' \underline{\Gamma}^j \sum_{i=1}^{12} \underline{e}_i \, \varepsilon_{t+1-i}^2. \tag{A6}$$

By the law of iterated expectations $E_t(\nu_{t+j}) = 0$ for all $j > 0$, and

$$E_t(\varepsilon^2_{t+j}) = \omega \sum_{i=0}^{j-1} \underline{e}_1{}' \, \underline{\Gamma}^i \, \underline{e}_1 + \underline{e}_1{}' \, \underline{\Gamma}^j \sum_{i=1}^{12} \underline{e}_i \, \varepsilon^2_{t+1-i}. \tag{A7}$$

Combining equations (A4) and (A7) we get

$$\mathrm{Var}_t(d_{t+j}) = \sum_{i=0}^{j-1} \psi_i^2 \left[\omega \sum_{h=0}^{j-i-1} (\underline{e}_1{}' \, \underline{\Gamma}^h \, \underline{e}_1) + \sum_{h=1}^{12} (\underline{e}_1{}' \, \underline{\Gamma}^{j-i} \, \underline{e}_h) \varepsilon^2_{t+1-h} \right]. \tag{A8}$$

This completes the derivation of the forecast formula used in the evaluation of equation (2.9) for $j \leq J$.

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Table 1

Real Monthly Percentage Returns
Short-Horizon Summary Statistics

	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
μ	.635	-	.503	.567	.458	.496	.670
	(.216)		[.636]	[.654]	[.624]	[.634]	[.694]
σ^2	33.5	-	.000	.000	.047	.025	.282
	(1.77)		[10.5]	[14.0]	[14.2]	[9.12]	[22.8]
b_3	.389	.000	.000	.000	.003	.893	.126
	(.091)	[.000]	[.097]	[.112]	[-5.35]	[.818]	[-1.32]
b_4	10.6	.000	.000	.000	.897	.259	.832
	(.182)	[3.00]	[2.99]	[2.99]	[75.2]	[6.76]	[16.1]
ρ_1	.116	.001	.000	.001	.023	.012	.020
	(.037)	[.000]	[-.002]	[-.004]	[-.018]	[-.003]	[-.012]
Q_{12}	32.3	.001	.002	.001	.023	.199	.499
	(4.90)	[11.3]	[11.6]	[11.1]	[10.4]	[19.4]	[32.2]
$\rho_1^{(2)}$	.289	.000	.000	.000	.007	.023	.013
	(.037)	[.000]	[-.001]	[-.003]	[.005]	[.077]	[.045]
$Q_{12}^{(2)}$	398.4	.000	.000	.000	.002	.064	.139
	(4.90)	[11.3]	[11.3]	[11.1]	[.896]	[172.1]	[190.2]

Note: The sample statistics are the mean, μ, the variance, σ^2, the skewness, b_3, the kurtosis, b_4, and the first-order autocorrelations for returns, ρ_1, and for squared returns, $\rho_1^{(2)}$. The Ljung-Box portmanteau tests for up to twelfth order serial correlation in the levels of returns and the squared returns are denoted Q_{12} and $Q_{12}^{(2)}$, respectively. The Data column gives the sample statistics with asymptotic standard errors constructed under the null hypothesis of i.i.d. normally distributed constant expected returns in parentheses. The column labelled Asymp. reports the p-values for this null hypothesis. The last five columns give the small sample or empirical p-values of the sample statistics from the five Monte Carlo experiments. The medians for the different empirical distributions are reported in square brackets.

Table 2

Real Monthly Returns
MA(1)-GARCH(1,1) Quasi-Maximum Likelihood Estimates

$$R_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

$$\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$$

	Data	Asymp	Null2	TVRP
μ	.778	-	.092	.002
	(.184)		[.643]	[.431]
θ	.101	-	.006	.030
	(.044)		[-.004]	[-.002]
ω	.668	-	.008	.383
	(.328)		[.233]	[.563]
α	.123	-	.357	.523
	(.023)		[.108]	[.127]
β	.861	-	.554	.435
	(.026)		[.865]	[.853]
b_3	-.547	1.00	1.00	.000
	(.091)	[.000]	[.555]	[-1.89]
b_4	4.39	.000	.499	1.00
	(.182)	[3.00]	[4.39]	[10.5]
Q_{12}	14.2	.288	.263	.396
	(4.90)	[11.3]	[11.0]	[12.6]
$Q_{12}^{(2)}$	11.8	.462	.984	.804
	(4.90)	[11.3]	[64.9]	[22.5]

Note: The data column reports the quasi-maximum likelihood estimates for the actual returns with robust asymptotic standard errors in parentheses. The summary statistics for the standardized residuals, $\hat{\varepsilon}_t\hat{\sigma}_t^{-1}$, are denoted as in Table 1. The Asymp. column reports the p-values in the asymptotic distribution under the null hypothesis of i.i.d. normally distributed standardized innovations. The Null2 and TVRP columns provide the p-values from the quasi-maximum likelihood estimates for the data generated under the two hypotheses. Medians are reported in square brackets.

Table 3

Real Monthly Returns
Multi-Period Regressions

$$\ln(R_{t+k,k}) = \alpha_{k,k} + \beta_{k,k}\ln(R_{t,k}) + u_{t+k,k},$$

Panel: A Traditional GMM Standard Errors

k	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
			Slope Coefficients				
1	.095	.162	.145	.158	.108	.175	.217
	(.068)	[.000]	[-.064]	[-.127]	[-.359]	[-.098]	[-.251]
12	-.068	.626	.649	.690	.741	.680	.730
	(.140)	[.000]	[-.244]	[-.511]	[-.801]	[-.324]	[-.713]
24	-.200	.000	.006	.014	.008	.012	.018
	(.044)	[.000]	[-.363]	[-.637]	[-.768]	[-.507]	[-1.14]
36	-.205	.162	.327	.368	.371	.362	.516
	(.146)	[.000]	[-.448]	[-.738]	[-.785]	[-.553]	[-1.31]
48	-.187	.408	.577	.610	.612	.590	.737
	(.227)	[.000]	[-.611]	[-.704]	[-.884]	[-.664]	[-1.40]
			$\chi^2(5)$ Statistics				
	15.8	.007	.237	.278	.311	.324	.460
		[4.35]	[8.61]	[9.77]	[10.6]	[11.1]	[14.3]
			Coefficients of Multiple Correlation, R^2				
1	.009		.014	.017	.049	.061	.102
			[.001]	[.001]	[.001]	[.001]	[.002]
12	.005		.535	.554	.661	.606	.712
			[.006]	[.006]	[.011]	[.008]	[.014]
24	.045		.169	.204	.217	.226	.378
			[.012]	[.013]	[.014]	[.015]	[.031]
36	.058		.228	.270	.235	.245	.410
			[.018]	[.021]	[.020]	[.021]	[.042]
48	.046		.353	.422	.385	.368	.512
			[.026]	[.032]	[.028]	[.024]	[.048]

Table 3 cont.

Panel B: GMM Standard Errors Calculated Under the Null Hypothesis

k	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
			Slope Coefficients				
1	.095	.171	.152	.166	.110	.172	.208
	(.069)	[.000]	[-.064]	[-.127]	[-.360]	[-.098]	[-.250]
12	-.068	.688	.691	.717	.675	.678	.748
	(.170)	[.000]	[-.232]	[-.446]	[-.481]	[-.253]	[-.587]
24	-.200	.411	.410	.443	.372	.414	.578
	(.244)	[.000]	[-.309]	[-.472]	[-.433]	[-.345]	[-.834]
36	-.205	.357	.380	.397	.344	.368	.504
	(.223)	[.000]	[-.331]	[-.538]	[-.487]	[-.372]	[-.836]
48	-.187	.276	.291	.335	.255	.289	.414
	(.172)	[.000]	[-.423]	[-.600]	[-.518]	[-.449]	[-.857]
			$\chi^2(5)$ Statistics				
	3.37	.644	.373	.380	.382	.341	.396
		[4.35]	[1.48]	[1.68]	[1.50]	[1.30]	[1.84]

Note: The Data column provides the sample statistics with asymptotic standard errors in parentheses. The Asymp. column reports the p-values in the asymptotic distributions for testing the null hypothesis $\beta_{k,k} = 0$. The remaining five columns provide the p-values from the empirical distributions for the sample statistics together with the medians in square brackets.

Table 4

Real Monthly Returns
One-Period Regressions

$$\ln(R_{t+1}) = \alpha_{1,2k-1} + \beta_{1,2k-1}\left(\sum_{j=1}^{2k-1} \omega_j \ln(R_{t+1-j})\right) + u_{t+1}$$

k	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
			Slope Coefficients				
1	.034	.504	.491	.512	.436	.523	.552
	(.052)	[.000]	[-.065]	[-.100]	[-.340]	[-.072]	[-.250]
12	-.201	.239	.260	.242	.187	.236	.319
	(.171)	[.000]	[-.249]	[-.402]	[-.465]	[-.287]	[-.593]
24	-.279	.283	.282	.311	.239	.251	.435
	(.260)	[.000]	[-.294]	[-.459]	[-.448]	[-.343]	[-.789]
36	-.153	.633	.641	.649	.605	.637	.760
	(.321)	[.000]	[-.349]	[-.510]	[-.464]	[-.359]	[-.819]
48	-.161	.631	.648	.667	.618	.638	.753
	(.336)	[.000]	[-.407]	[-.580]	[-.489]	[-.407]	[-.875]
			$\chi^2(5)$ Statistics				
	2.85	.723	.731	.762	.732	.737	.851
		[4.35]	[4.57]	[4.79]	[4.58]	[4.58]	[5.74]
			Coefficients of Multiple Correlation, R^2				
1	.001		.403	.427	.495	.533	.586
			[.001]	[.001]	[.001]	[.001]	[.001]
12	.005		.183	.172	.269	.262	.369
			[.001]	[.001]	[.001]	[.001]	[.002]
24	.003		.258	.277	.271	.289	.462
			[.001]	[.001]	[.001]	[.001]	[.002]
36	.001		.587	.596	.572	.590	.712
			[.001]	[.001]	[.001]	[.001]	[.001]
48	.000		.634	.649	.589	.634	.733
			[.001]	[.001]	[.001]	[.001]	[.001]

Note: See Table 3. The weights ω_j are defined after equation (4.9).

Table 5

Real Monthly Stock Prices and Dividends

Panel A: Unit Root Tests

$$\Delta u_t = \mu + \rho u_{t-1} + \phi_1 \Delta u_{t-1} + \ldots + \phi_{12} \Delta u_{t-12} + e_t$$

$t_{\rho=0}$	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
p_t	-1.83	.639 [-1.56]	.652 [-1.53]	.598 [-1.62]	.533 [-1.78]	.627 [-1.51]	.425 [-1.98]
d_t	-1.35	.403 [-1.56]	.429 [-1.54]	.429 [-1.54]	.429 [-1.54]	.437 [-1.50]	.419 [-1.54]

Panel B: Cointegration Tests

$$p_t = a + bd_t + u_t$$

	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
a	3.94	-	1.00 [5.26]	.991 [5.25]	.962 [5.21]	1.00 [5.26]	.947 [5.28]
b	1.31	-	.000 [1.00]	.001 [1.00]	.002 [.954]	.001 [1.00]	.024 [.994]
$t_{\rho=0}$ $\mu=0$	-4.57	.999 [-2.21]	.000 [-7.25]	.954 [-3.39]	.952 [-3.55]	.054 [-6.51]	.718 [-4.06]
$t_{\rho=0,\mu\neq0}$ a=0 b=1	-3.87	.999 [-1.56]	.000 [-7.23]	.848 [-3.17]	.794 [-3.40]	.029 [-6.42]	.499 [-3.95]

Note: Augmented Dickey-Fuller t-tests for a unit root. In Panel B, the coefficients a and b denote the OLS estimates from the cointegrating regression. The statistics $t_{\rho=0}$ give the t-tests for a unit root in the regression residuals; i.e., the null hypothesis of no cointegration. The row labelled $t_{\rho=0,\mu\neq0}$ denotes the t-test for a unit root in p_t - d_t; i.e., imposing a=0 and b=1. The Data column provides the sample statistics. The Asymp. column reports the p-values in the simulated asymptotic distributions. The last five columns provide the p-values from the Monte Carlo experiments under the different hypotheses. Medians are reported in square brackets.

Table 6

Volatility Tests

Panel A: Second Moment Volatility Tests

$$q_{t+k} \equiv \left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2 - \left(\frac{P_t^{*k}}{P_t} - 1\right)^2 - \left(1 - \frac{P_t^0}{P_t}\right)^2.$$

k	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
1	-.007	.001	.000	.000	.041	.008	.346
	(.002)	[.000]	[-.001]	[-.002]	[-.002]	[-.001]	[-.005]
12	-.032	.000	.000	.000	.173	.013	.142
	(.003)	[.000]	[-.002]	[-.003]	[-.006]	[-.001]	[-.009]
24	-.059	.023	.000	.000	.009	.000	.331
	(.026)	[.000]	[-.001]	[-.007]	[-.010]	[-.001]	[-.034]
36	-.082	.008	.000	.000	.012	.000	.307
	(.031)	[.000]	[-.001]	[-.010]	[-.014]	[-.001]	[-.041]
48	-.112	.006	.000	.000	.021	.003	.266
	(.041)	[.000]	[-.001]	[-.012]	[-.018]	[-.001]	[-.046]

Table 6 cont.

Panel B: Diagnostics on Ex-Post Rational Prices and Dividend Yields

Hor.	Data	Asymp	Null1	Fads	Bubble	Null2	TVRP
Second Moment of Ex-Post Rational Price Relative to Naive Price							
1	.142		.000	.000	.093	.008	.180
	(.023)		[.005]	[.016]	[.030]	[.004]	[.047]
12	.117		.000	.000	.017	.003	.187
	(.020)		[.004]	[.013]	[.024]	[.003]	[.041]
24	.155		.000	.000	.114	.015	.140
	(.028)		[.023]	[.036]	[.049]	[.022]	[.056]
36	.144		.000	.000	.118	.017	.153
	(.028)		[.033]	[.042]	[.055]	[.028]	[.061]
48	.141		.000	.000	.115	.030	.155
	(.029)		[.040]	[.048]	[.061]	[.033]	[.063]
Second Moment of Ex-Post Rational Price Relative to Unity							
1	.003		.000	.000	.045	.027	.274
	(.001)		[.001]	[.001]	[.001]	[.001]	[.002]
12	.003		.000	.000	.197	.021	.166
	(.001)		[.001]	[.001]	[.001]	[.001]	[.001]
24	.069		.000	.000	.083	.023	.295
	(.016)		[.021]	[.027]	[.027]	[.018]	[.042]
36	.080		.000	.000	.085	.035	.316
	(.018)		[.029]	[.036]	[.038]	[.024]	[.052]
48	.107		.000	.000	.074	.033	.269
	(.028)		[.035]	[.044]	[.047]	[.030]	[.061]
Second Moment of Naive Price Relative to Unity							
	.146		.000	.000	.092	.008	.184
	(.023)		[.005]	[.016]	[.031]	[.004]	[.049]

Note: Mankiw, Romer and Shapiro (1991) volatility tests. The ex-post rational price is denoted P_t^{*k}, and the naive price is P_t^0. The results reported in Panel A test the mean of q_{t+k} equal to zero. The three second-moment components of q_{t+k} are reported in Panel B. The columns report the p-values for the sample statistics with medians in square brackets.

Figure 1

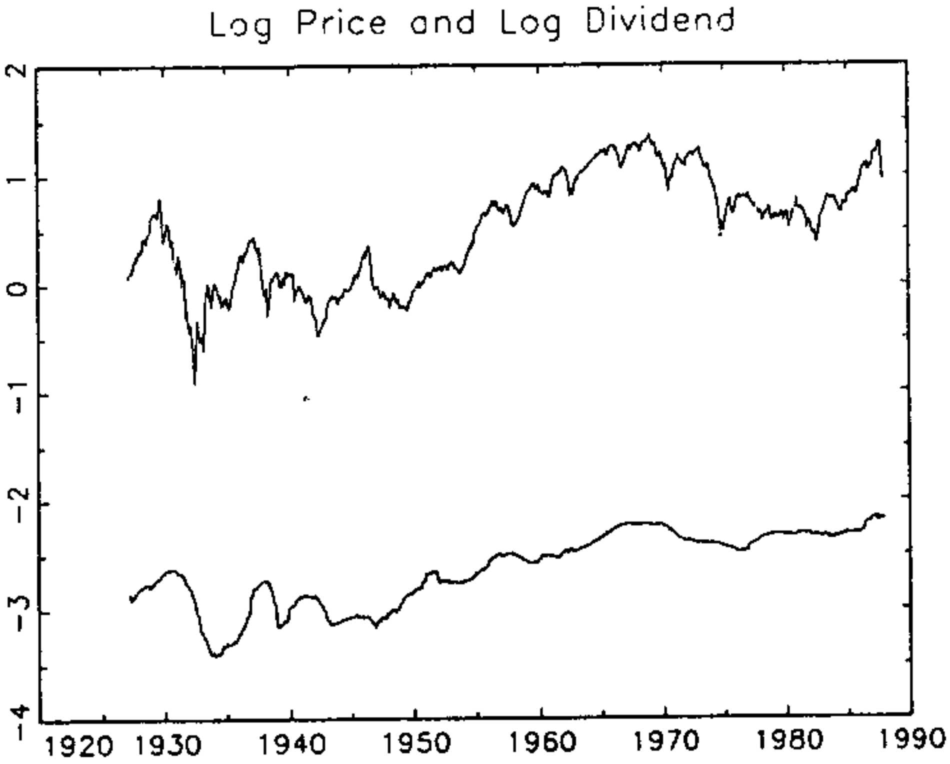


Figure 2

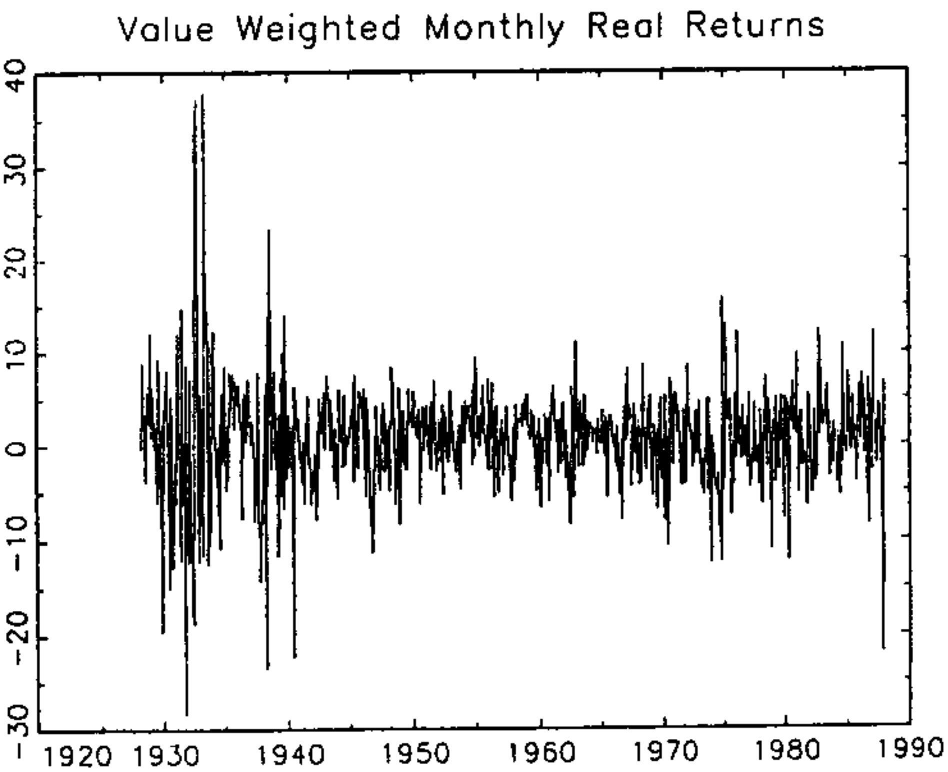
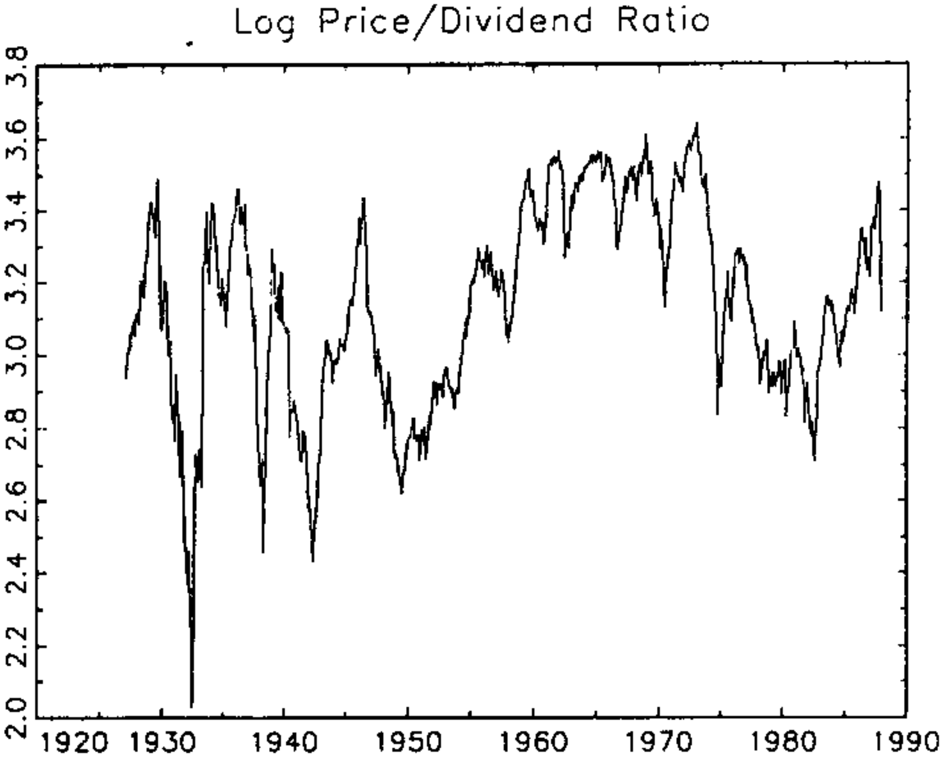


Figure 3


Log Price/Dividend Ratio
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1920
1930
1940
1950
1960
1970
1980
1990