THE RANDOM WALK HYPOTHESIS, PORTFOLIO ANALYSIS AND THE BUY-AND-HOLD CRITERION**

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I. Introduction

Recent literature has witnessed the emergence of an impressive body of empirical evidence relating to the relationship which exists between successive security price changes. The great majority of this evidence has tended to support what has come to be known as the theory of random walks in security prices--that is, the theory that successive security price changes behave as independent random variables, which implies that knowledge of "the past history of a series of price changes cannot be used to predict future changes in any 'meaningful' way."[1]

For purposes of empirical investigation the term "meaningful" has come to mean that knowledge of the past performance of a security or securities cannot be used to increase expected returns to the investor. Rather, if the theory of random walks holds, such knowledge of past performance should not on the average tend to increase expected returns above those which could be obtained if the investor employed a naive buy-and-hold strategy. That is, there should be no trading rules which will consistently produce greater expected returns than that rule which calls for the investor to simply purchase a security and hold it.

When the empirical investigation is concerned only with trading rules applied to individual securities, the above criterion tends to provide an adequate standard of comparison. However, when the investigation is concerned with portfolios of securities this criterion is not sufficient--that

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[1]Eugene F. Fama and Marshall E. Blume, "Filter Rules and Stock Market Trading," *Journal of Business*, Volume 39, No. 1, Part II (January 1966), p. 226.

is, the buy-and-hold strategy is no longer the appropriate standard against which to measure the performance of alternative policies. It will be shown that a mechanical trading rule exists which, when applied to portfolios of securities, consistently leads to significantly greater expected returns than those produced by the naive buy-and-hold strategy. Further, it will be shown that when applied to individual securities, this rule leads to precisely the same expected returns as those produced by the naive buy-and-hold strategy. Finally, it will be shown that the nature of this rule is such that its existence in no way refutes the argument that successive security price changes behave as independent random variables, but rather supports this position.

II. The Trading Rules

The investment strategies or trading rules with which the following analysis will be concerned are (1) the "buy-and-hold" strategy and (2) what will be termed the "fixed investment proportion maintenance" strategy. The naive buy-and-hold strategy may be defined as the selection and purchase of a security or group of securities (portfolio) at time t with the sale of the security or group of securities at time t + i (for i = 1 to n; where n is some predetermined number of sub-periods comprising the "holding period"). During the holding period, it is assumed that all dividends are reinvested in the particular security on which they were paid. For example, if securities x and y comprise a portfolio, then all dividends paid on security x are reinvested in security x and those paid on security y are reinvested in security y.[2]

The fixed investment proportion maintenance strategy may be defined as the selection and purchase of a security or group of securities (portfolio)

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[2]Theoretically, whether we deal with one or m securities should be of no consequence in testing the validity of the random walk hypothesis since sums of independent random variables are themselves independent random variables, and therefore, the successive "price" changes of the aggregate security (portfolio) will tend to behave as an independent random variable if the successive price changes of the component securities behave as independent random variables over time. The fact that the securities are correlated among themselves is of no direct consequence to this analysis in that this correlation affects only the variability of the aggregate securities expected return and not the expected return itself.

at time t, with the sale of the security or group of securities at time t + i (for i = 1 to n; where n is some predetermined number of sub-periods comprising the "holding period"). During this holding period it is again assumed that all dividends are reinvested, however, not necessarily in the security on which they were paid. Rather, it is assumed that the investor reallocates the investment bundle at the end of each sub-period i among the m securities included in the portfolio such that the same proportion of the investment bundle is maintained in each of the m securities as was originally allocated at time t.[3] For example, if at time t 50 percent of some given investment bundle is allocated to each of two securities, the above trading rule dictates that at the end of each sub-period i in the holding period, the investment bundle (including dividends) be reallocated such that the proportion invested in each security is maintained at 50 percent.

The reader will readily see that if the alternative strategies are applied to an individual security, the returns yielded will be identical. However, if applied to portfolios of securities, the returns should tend to differ since under the fixed proportion maintenance strategy the investor will periodically be led to purchase more shares of securities whose prices have fallen, and generally to sell shares of securities whose prices have risen. Such a strategy should tend to yield returns superior to the naive buy-and-hold strategy if (1) there exists some intrinsic value of a security about which the market value fluctuates randomly and (2) the market is characterized by an inherent upward tendency over the long run.[4]

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[3]For the purposes of this analysis, all empirical work is based upon the assignment of equal proportions of the investment bundle to each security. It is quite possible that some other initial assignment may lead to differing results, however, this possibility is not examined here.

[4]As was pointed out to the author by Professor William F. Sharpe of the School of Social Sciences, University of California, Irvine; if it is assumed that the risk of a security (as measured by, say, the variance of its return) is stable over time, the fixed investment proportion maintenance strategy will tend to be superior to the buy-and-hold strategy, even if both yield identical returns. This results from the fact that the former maintains the extent of diversification of a portfolio over time, and therefore its risk level, whereas the latter leads to decreased diversification over time, and therefore, to generally increased risk. This reduction in the extent of diversification over time under the buy-and-hold strategy results from the fact that the portfolio will include smaller proportions of securities whose prices have fallen and larger proportions of securities whose prices have risen.

III. The Data

The data employed in this analysis were obtained from several sources. First, "actual" data were obtained on 470 securities listed in the Standard & Poor's Index for the year 1958. Observations on each security were taken at semi-annual intervals for the period January 1, 1958 - July 1, 1967[5] yielding 20 observations for each security. From these data, value relatives were computed for each security and period as follows: Where p_i^k is the price of security k at time i, p_{i+1}^k is the price of security k at time i + 1 and d_i^k is the dividend paid on security k during the interval from time i to time i + 1.[6]

$$R_i^k = \frac{p_{i+1}^k + d_i^k}{p_i^k} \; ; \text{ for } \begin{matrix} i = 1 \text{ to } 19 \\ k + 1 \text{ to } 470 \end{matrix}$$

In all, 8,930 value relatives were obtained (19 x 470), providing the basis for the analysis.

The second source of data was what may be termed "Random Actual" observations. These data were obtained by randomly rearranging the 8,930 actual value relatives. This process yielded a distribution of value relatives from which we randomly drew sets of 19 observations to simulate one security's experience. The purpose in obtaining these data was to compare the performance of actual securities with the performance of hypothetical securities whose price changes were random in nature and were distributed as the actual price changes.

The third source of data was what may be termed "Random Normal" observations. These data were obtained from the random number generator of the IBM 7094 computer at the University of Washington. A process was defined such that value relatives were randomly generated from a population whose parameters were equal to those of the sample of "actual" value relatives. The purpose in obtaining these data was to compare the performance of the actual

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[5]The 1958 Index contains 500 securities. Satisfactory data were available on 470 of these. All values were adjusted for stock splits and stock dividends during the period.

[6]The reader will note that R_i^k is simply the rate of return plus 1. This form was selected since all computations in the later analysis were made in terms of logarithms.

securities, as well as the distributions of their returns, with those given by securities generated from randomly selected, normally distributed value relatives.

Given these data, the analysis was conducted with the primary purpose of defining a more adequate standard than that provided by the buy-and-hold strategy, against which to measure the performance of alternative trading rules with respect to portfolios. Secondary purposes were to add to the available evidence pertaining to the accuracy of the theory of random walks in describing security price changes as well as to the nature of the distributions of these price changes.

IV. The Analysis

Given the three sets of data described above--that is, the actual, random actual, and random normal sets--portfolios ranging in size from 1 to 40 securities were selected from each set under the alternative trading strategies as follows: under the buy-and-hold strategy the geometric mean return was computed for each of the 470 available securities over the 19 sub-periods as below

$$R_B^k = \exp\left(\frac{1}{n} \cdot \sum_{i=1}^{n} \text{Log}_e R_i^k\right)$$

where R_B^k is the return on security k under the buy-and-hold strategy and R_i^k is the value relative for security k in sub-period i (for i = 1 to n; n = 19). These became observations of portfolios of one security. Next, two securities were selected at random from among the 470 and 469 available, respectively,[7] and the average geometric mean return for this combination computed as below

$$\bar{R}_{B2} = \frac{1}{m} \cdot \sum_{\ell=1}^{m} R_B^{\ell}$$

[7]The size of the population from which the samples were drawn was considered large enough, and the samples small enough, such that sampling without replacement would not significantly affect the accuracy of the results.

where $\bar{R}_{B2}$ is the return on the portfolio with 2 securities under the buy-and-hold strategy, R_B^{ℓ} is the geometric mean return on security ℓ over the 19 sub-periods for the buy-and-hold strategy and m is the number of securities in the portfolio (here 2). These became observations of portfolios with 2 securities. The process was continued for 3, 4, ... , 40 securities, with the portfolio geometric mean return under a buy-and-hold strategy computed for each resulting portfolio.[8] As a result, each run generated 40 portfolios ranging in size from 1 to 40 securities (see Table 1). In all, there were 600 such runs resulting in 23,870 portfolios (39 x 600 plus 470 of size 1).[9]

The process described above was performed on all three sets of data such that 23,870 portfolios were obtained for each data set under the buy-and-hold strategy.

The reader will note that the return under a buy-and-hold strategy is simply the average of the geometric mean returns for each of the individual securities included in the portfolio. Computing the portfolio return in this manner satisfies the condition that dividends be reinvested in the securities on which they were obtained.

Under the Fixed Investment Proportion Maintenance (FIPM) strategy, the process was initially the same as under the buy-and-hold strategy: The geometric mean return was computed for each of the 470 available securities over the 19 sub-periods as below

$$R_D^k = \exp\left(\frac{1}{n} \cdot \sum_{i=1}^{n} \text{Log}_e R_i^k\right)$$

where R_D^k is now the return on security k under the FIPM strategy and R_i^k is again the value relative for security k in sub-period i (for i = 1 to n;

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[8]Note that once a security was selected in a particular run, this security was considered ineligible for further consideration in this run. As a result, each of the 40 portfolios selected is in and of itself independent from the others.

[9]This number was derived as a result of the fact that only 470 securities are available in the sample and therefore only 470 portfolios of size 1 were obtained rather than 600.

Table 1

The following describes in tabular form the portfolio selection process under the buy-and-hold strategy for one computer run:

Securities selected (by identification number)	1	2	3	. . .	4
1	R_B^{113}	R_B^{44}	R_B^{11}		R_B^{244}
2		R_B^{219}	R_B^{269}		R_B^{88}
3			R_B^{98}		R_B^{37}
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40					R_B^{222}
Mean Portfolio Return ($\bar{R}_B$)	.117	.087	.071		.0608

NOTE: The mean portfolio return above is the "geometric mean semi-annual return."

n = 19). Note that this yields precisely the same return for a single security as was derived for portfolios of size 1 under the buy-and-hold strategy. At this point, however, the processes diverge. For portfolios of size 2, two securities were again selected <u>at random</u> from among the 470 and 469 available, respectively, and the portfolio geometric mean return for this combination computed as below: First, the portfolio return for each sub-period was computed as

$$\bar{R}_i = \frac{1}{m} \cdot \sum_{k=1}^{m} R_i^k$$

where $\bar{R}_i$ is the portfolio return in period i. Second, the portfolio return was computed over the 19 sub-periods,

$$\bar{R}_{D2} = \exp\left(\frac{1}{n} \cdot \sum_{i=1}^{n} \text{Log}_e \bar{R}_i\right)$$

where $\bar{R}_{D2}$ is the portfolio geometric return and $\bar{R}_i$ is the portfolio return in sub-period i (i = 1 to 19). These became observations of portfolios of two securities under the FIPM strategy. This process was continued for 3, 4, ... , 40 securities as before, and again 600 such runs were made yielding 23,870 portfolios.[10]

In total then, 143,220 portfolios were generated--23,870 for each set of data under each of the two strategies (6 x 23,870).[11]

The portfolio returns derived from the above process were next segregated by data set and strategy combination, then sorted in ascending order for each portfolio size (1 to 40) and the following summary measures recorded: (1) the minimum return, (2) the tenth percentile return, (3) the

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[10]The careful reader will note that computation of the portfolio return as shown for the FIPM strategy yields a return which is as if the investor had reallocated his investment bundle at the end of each sub-period such that equal proportionate amounts were maintained in each security included.

[11]It should be pointed out that the same random numbers were employed to select the portfolios for each data set--strategy combination such that the only variable factor in the analysis was the strategy employed.

first quartile return, (4) the 50th percentile return (median), (5) the mean return, (6) the 3rd quartile return, (7) the 90th percentile return, and (8) the maximum return (see Table 2).

The next step in the analysis was to compute the overall average values over all portfolio sizes (1 to 40), as well as the average values over portfolio sizes 10 to 40, for each of the above summary measures on all data sets--strategy combinations so that comparison of results could be made (see Tables 3A and 3B).[12] It is readily apparent that significant differences exist between the average values of the summary measures for the respective data sets under the two alternative strategies (see Table 4)[13] and that these differences all indicate superior performance by the portfolios employing the FIPM strategy. The reader will note that the FIPM strategy was not only superior to the buy-and-hold strategy with respect to the "actual" data but also with respect to the "random actual" and "random normal" sets. Further, the close correspondence of the magnitudes of this superiority across the three data sets suggests that the "actual" observations correlate rather closely with those obtained on the two "random" sets--especially under the buy-and-hold strategy--which would tend to be the case if the "actual" data was itself random in nature. On the other hand, close examination of Tables 3A and 3B indicate that under both strategies, the actual data yielded generally lower values than either of the sets of random data,[14] indicating that some degree of non-randomness exists in the actual data. However, referring to the initial statement on the theory of random walks, this

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[12]The average values over sizes 10 to 40 were computed since it has been shown that, on the average, diversification eliminates more than 97 percent of the unsystematic portion of a portfolio's return for portfolios with 10 or more securities, and therefore, such an average may tend to yield a more accurate and representative value than one which included both unsystematic and systematic portions of return. See John L. Evans and Stephen H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," *Journal of Finance*, Volume 23, No. 5 (December 1968), forthcoming.

[13]These differences among the respective pairs of returns were all found to be significant at the .01 level.

[14]Exceptions to this tendency are found under the buy-and-hold strategy for the "actual" and "random normal" data sets.

Table 2*

The following are the values obtained for the distribution of portfolio returns using actual data and the buy-and-hold strategy for portfolio size 1, 10, 20, and 40:

Portfolio Size	Min.	P_{10}	Q_1	P_{50}	$\bar{X}$	Q_3	P_{90}	Max.
1	.9096	1.0204	1.0412	1.0628	1.0652	1.0883	1.1115	1.2278
10	1.0135	1.0519	1.0577	1.0661	1.0656	1.0725	1.0803	1.1049
20	1.0364	1.0557	1.0601	1.0656	1.0657	1.0711	1.0761	1.0858
40	1.0422	1.0575	1.0615	1.0653	1.0653	1.0689	1.0728	1.0859

Min. = The minimum average semi-annual geometric mean return for given portfolio sizes

P_{10} = 10th percentile return

Q_1 = First quartile return

P_{50} = Median return

Q_3 = Third quartile return

P_{90} = 90th percentile return

Max. = Maximum return

*NOTE: The table describes the distribution of portfolio returns derived from 600 rates of return for each portfolio size shown (1, 10, 20, and 40).

non-randomness does not appear to be of significant magnitude or direction to be considered "meaningful" as we have defined the term--that is, in such a manner as could be utilized to increase expected profits. In fact, the results suggest that investors would be better off if true randomness did exist.

Another result indicating the superiority of the FIPM strategy is the direction of the skewness of the distributions of portfolio returns (see Tables 3A and 3B). Under the FIPM strategy the distributions of returns obtained from the actual and random actual distributions are positively skewed, whereas under the buy-and-hold strategy the distributions of returns for these data sets are weak positive and negative, respectively. This implies that there is a larger probability of relatively low returns using a buy-and-hold strategy and a larger probability of relatively high returns using a FIPM strategy over and above what is indicated by the respective mean values.

Finally, the reader will note that the range of portfolio returns from the actual data is appreciably less than the ranges of portfolio returns from either the random actual or random normal--that is, the distribution of actual portfolio returns is "tighter" than those of random actual or random normal portfolio returns. It would seem that if the distribution of the actual value relatives was of the infinite variance variety this would not tend to be the case. However, as a result of the fact that these distributions are cross-sectional and based upon "average" values, this finding is of questionable significance.

V. Conclusions

The principle conclusion to be drawn from this analysis is that irrespective of the degree of randomness or "form" characterizing the empirical distribution of security price changes, employment of the FIPM strategy (or rule) yields returns which are significantly superior to those yielded by a naive buy-and-hold strategy. Further, the magnitude of the difference in returns is such that this superiority is maintained even after considerations of transactions costs are made--which will be of negligible importance assuming reinvestment of dividends under the buy-and-hold strategy. As a result, it appears highly reasonable to assert that one should replace the

Table 3A

The following are the average semi-annual values for selected summary measures over all portfolio sizes for each data set--strategy combination:

	FIPM			BUY-AND-HOLD		
	ACT.	R.A.	R.N.	ACT.	R.A.	R.N.
Minimum Return	1.0430	1.0386	1.0391	1.0293	1.0189	1.0198
Median	1.0791	1.0853	1.0839	1.0654	1.0660	1.0637
Mean	1.0795	1.0855	1.0837	1.0654	1.0659	1.0634
Maximum Return	1.1233	1.1359	1.1283	1.1004	1.1115	1.1059
Range	.0803	.0973	.0892	.0711	.0926	.0861
Range + Tail	.0438	.0494	.0446	.0350	.0456	.0425
Range - Tail	.0365	.0479	.0446	.0361	.0470	.0436
Skewness	.2330	.0715	.0151	.0045	-.0273	-.0318

KEY:
ACT. = Actual distribution
R.A. = Random actual distribution
R.N. = Random normal distribution
Skewness = Average moment coefficient of skewness

Table 3B

The following are the average semi-annual values for selected summary measures over portfolio sizes 10-40 for each data set--strategy combination:

	FIPM			BUY-AND-HOLD		
	ACT.	R.A.	R.N.	ACT.	R.A.	R.N.
Minimum Return	1.0531	1.0517	1.0509	1.0387	1.0318	1.0302
Median	1.0802	1.0867	1.0852	1.0654	1.0660	1.0636
Mean	1.0805	1.0869	1.0851	1.0653	1.0658	1.0635
Maximum Return	1.1125	1.1245	1.1192	1.0904	1.0997	1.0956
Range	.0594	.0728	.0683	.0517	.0679	.0654
Range + Tail	.0320	.0376	.0341	.0251	.0339	.0321
Range - Tail	.0274	.0352	.0342	.0266	.0340	.0333
Skewness	.2180	.0878	.0247	-.0123	-.0314	-.0386

KEY:
ACT. = Actual distribution
R.A. = Random actual distribution
R.N. = Random normal distribution
Skewness = Average moment coefficient of skewness

Table 4

The following represent the annualized differences between the buy-and-hold and FIPM 10-40 average summary measures for the three data sets, positive values indicate differences in favor of the FIPM strategy:

	ACTUAL	R.A.	R.N.
Minimum Return	.0288	.0398	.0414
Median	.0296	.0414	.0432
Mean	.0304	.0422	.0432
Maximum Return	.0442	.0496	.0472
Range	.0154	.0098	.0058

buy-and-hold strategy with the FIPM strategy as the appropriate standard against which to measure the performance of mutual fund portfolios and portfolios in general.[15]

A further conclusion suggested by the results is that although some degree of non-randomness exists in the distribution of security price changes, it is not of a magnitude that should be considered "meaningful" to the investor dealing with "portfolios" of securities.

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[15]The reader should note that modification of the suggested strategy allowing for additions to or deductions from the initial investment bundle over time have no effect on the results shown above so long as these additions or deductions comply with the "proper maintenance" criterion.

REFERENCES

Evans, John L. and Stephen H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," *Journal of Finance*, Volume 23, No. 5 (December 1968), forthcoming.

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