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ON THE EFFICIENCY OF COMPETITIVE STOCK MARKETS WHERE TRADES HAVE DIVERSE INFORMATION

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1. INTRODUCTION

I HAVE SHOWN elsewhere that competitive markets can be "over-informationally" efficient. (See Grossman [1975] for this and a review of other work in this area.) If competitive prices reveal too much information, traders may not be able to earn a return on their investment in information. This was demonstrated for a market with two types of traders, "informed" and "uninformed." "Informed" traders learn the true underlying probability distribution which generates a future price, and they take a position in the market based on this information. When all informed traders do this, current prices are affected. "Uninformed" traders invest no resources in collecting information, but they know that current prices reflect the information of informed traders. Uninformed traders form their beliefs about a future price from the information of informed traders which they learn from observing current prices.

In the above framework, prices transmit information. However, it is often claimed that prices aggregate information. In this paper we analyze a market where there are n-types of informed traders. Each gets a "piece of information." In a simple model we study the operation of the price system as an aggregator of the different pieces of information.

We consider a market where there are two assets; a risk free asset and a risky asset. Each unit of the risky asset yields a return of $\tilde{P}_1$ dollars. $\tilde{P}_1$ will also be referred to as the price of the risky asset in period 1. In period 0 (the current period), each trader gets information about $\tilde{P}_1$ and then decides how much of risky and non-risky assets to hold. This determines a current price of the risky asset, P_0, which will depend on the information received by all traders. We assume that the ith trader observes y_i, where $y_i = P_1 + \epsilon_i$. There is a noise term, ϵ_i, which prevents any trader from learning the true value of P_1. The current equilibrium price is a function of $(y_1, y_2, \ldots, y_n)$; write it as $P_0(y_1, y_2, \ldots, y_n)$.

The main result of this paper is that when there are n-types of traders ($n > 1$), P_0 reveals information to each trader which is of "higher quality" than his own information. That is, the competitive system aggregates all the market's information in such a way that the equilibrium price summarizes all the information in the

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market. $P_0(y_1,y_2,\ldots,y_n)$ is a sufficient statistic for the unknown value of P_1. A trader who invests nothing in information and observes the market price can achieve a utility as high as traders who pay for the information y_i. Similarly, a trader who purchases y_i and then observes $P_0(y)$ (where $y \equiv (y_1,y_2,\ldots,y_n)$), finds that y_i is redundant; $P_0(y)$ contains all the information he requires. That is, informationally efficient price systems aggregate diverse information perfectly, but in doing this the price system eliminates the private incentive for collecting the information.

The above result is demonstrated in the context of a simple mean-variance model. The result that the price system perfectly aggregates information is not robust. This is shown in the context of the above model when "noise" is added. One example of "noise" is an uncertain total stock of the risky asset. However, the paradoxical nature of "perfect markets," which the model illustrates, is robust. When a price system is a perfect aggregator of information it removes private incentives to collect information. If information is costly, there must be noise in the price system so that traders can earn a return on information gathering. If there is no noise and information collection is costly, then a perfect competitive market will break down because no equilibrium exists where information collectors earn a return on their information, and no equilibrium exists where no one collects information. The latter part follows from the fact that if no one collects information then there is an incentive for a given individual to collect costly information because he does not affect the equilibrium price. When many individuals attempt to earn a return on information collection, the equilibrium price is affected and it perfectly aggregates their information. This provides an incentive for individuals to stop collecting information. In Grossman [1975] there is a more detailed analysis of the breakdown of markets when price systems reveal too much information.

On the other hand, when there is noise so that the price system does not aggregate information perfectly, the allocative efficiency properties of a competitive equilibrium may break down. Hayek [1945] argues that the essence of a competitive price system is that when a commodity becomes scarce its price rises and this induces people to consume less of the commodity and to invest more in the production of the commodity. Individuals need not know why the price has risen, the fact that there is a higher price induces them to counteract the scarcity in an efficient way. This argument breaks down when the price system is noisy. We will show that in such cases each individual wants to know why the price has risen (i.e., what exogenous factors make the price unusually high), and that an optimal allocation of resources involves knowing why the price has risen (i.e., knowledge of the states of nature determining current prices is required).

2. The Model

Assume that trader "i" has an initial wealth W_{0i}. Using W_{0i}, he can purchase two assets; a risk free asset and a risky asset. His wealth in period 1, W_{1i} is given by

$$\tilde{W}_{1i} = (1+r)X_{Fi} + \tilde{P}_1 X_i, \tag{1}$$

where X_{Fi} is the value of risk free assets purchased in period 0, X_i is the number of units of risky assets purchased in period 0, $r \geqslant 0$ is the exogenous rate of return on the risk free asset, and $\tilde{P}_1$ is the (unknown) exogenous payoff per unit on the risky asset (also called the period 1 price of the risky asset). The budget constraint is

$$W_{0i} = X_{Fi} + P_0 X_i, \tag{2}$$

where P_0 is the current price of the risky asset. Substituting (2) into (1) to eliminate X_{Fi} yields:

$$\tilde{W}_{1i} = (1+r)W_{0i} + \left[\tilde{P}_1 - (1+r)P_0\right]X_i. \tag{3}$$

At time zero, P_1 is unknown. The ith trader observes y_i, where

$$y_i = P_1 + \epsilon_i, \tag{4}$$

and P_1 is a realization of the random variable $\tilde{P}_1$. Thus, a fixed, but unknown, realization of $\tilde{P}_1$ mixes with noise, ϵ_i, to produce the observed y_i. Later, we shall argue that traders also get information from P_0. For the present, let I_i denote the information available to the ith trader. Assume that the ith trader has a utility function

$$U_i(\tilde{W}_{1i}) = -e^{-a_i \tilde{W}_{1i}}, \qquad a_i > 0, \tag{5}$$

where a_i is the coefficient of absolute risk aversion. Each trader is assumed to maximize the expected value of $U_i(\tilde{W}_{1i})$ conditional on I_i. If $\tilde{W}_{1i}$ is normally distributed conditional on I_i, then

$$E\left[U_i(\tilde{W}_{1i}) \mid I_i\right] = -\exp\left\{-a_i\left[E\left[\tilde{W}_{1i} \mid I_i\right] - \frac{a_i}{2}\operatorname{Var}\left[\tilde{W}_{1i} \mid I_i\right]\right]\right\}, \tag{6}$$

where $\operatorname{Var}[\tilde{W}_{1i} \mid I_i]$ is the conditional variance of $\tilde{W}_{1i}$ given I_i. It follows that to maximize $E[U_i(\tilde{W}_{1i}) \mid I_i]$ is equivalent to maximizing

$$E\left[\tilde{W}_{1i} \mid I_i\right] - \frac{a_i}{2}\operatorname{Var}\left[\tilde{W}_{1i} \mid I_i\right], \tag{7}$$

since the expression in (7) in a monotone increasing transformation of the expression in (6). All we have shown is that mean-variance analysis in the Normal case can be derived from the utility function in (5).

From (3),

$$E\left[\tilde{W}_{1i} \mid I_i\right] = (1+r)W_{0i} + \left\{E\left[\tilde{P}_1 \mid I_i\right] - (1+r)P_0\right\}X_i \tag{8}$$

and

$$\operatorname{Var}\left[\tilde{W}_{1i} \mid I_i\right] = X_i^2 \operatorname{Var}\left[\tilde{P}_1 \mid I_i\right]. \tag{9}$$

In deriving (8) and (9) we have used the fact that W_{0i}, r, and P_0 are known to the firm in period 0. Thus, from (7)–(9), the consumer's problem is to maximize

$$(1+r)W_{0i} + \left\{E\left[\tilde{P}_1 \mid I_i\right] - (1+r)P_0\right\}X_i - \frac{a_i}{2}X_i^2 \operatorname{Var}\left[\tilde{P}_1 \mid I_i\right] \tag{10}$$

by choosing X_i. Using the calculus, an optimal X_i, X_i^d, satisfies

$$X_i^d = \frac{E\left[\tilde{P}_1 \mid I_i\right] - (1+r)P_0}{a_i \operatorname{Var}\left[\tilde{P}_1 \mid I_i\right]}. \tag{11}$$

Thus, the demand for the risky asset depends on its expected price appreciation and on its variance. Let $\overline{X}$ be the total stock of the risky asset. An equilibrium price in period 0 must cause $\sum_{i=1}^{n} X_i^d = \overline{X}$. From (11), the ith trader's demand for the risky asset depends on the information he receives. This depends on the observation he gets, y_i. Thus, since the total demand for the risky asset depends on $y_1, y_2, \ldots, y_n$, it is natural to think of the market clearing price as depending on the y_i, $i = 1, 2, \ldots, n$. Let $y \equiv (y_1, y_2, \ldots, y_n)$, then the equilibrium price is some function of y, $P_0(y)$. That is, different information about the return on an asset leads to a different equilibrium price of the asset.

There are many different functions of y. For a particular function, $P_0^*(y)$ to be an equilibrium we require that: for all y,

$$\sum_{i=1}^{n} \left\{ \frac{E\left[\tilde{P}_1 \mid y_i, P_0^*(y)\right] - (1+r)P_0^*(y)}{a_i \operatorname{Var}\left[\tilde{P}_1 \mid y_i, P_0^*(y)\right]} \right\} = \overline{X}. \tag{12}$$

(12) states that the total demand for the risky asset must equal the total supply for each y. (Throughout we put no non-negativity constraint on prices. By proper choice of parameters the probability of a negative price can be made arbitrarily small.) The ith trader's demand function under the price system $P_0^*(y)$ is

$$X_i^d\left[P_0^*, y_i\right] = \frac{E\left[\tilde{P}_1 \mid y_i, P_0^*(y)\right] - (1+r)P_0^*}{a_i \operatorname{Var}\left[\tilde{P}_1 \mid y_i, P_0^*(y)\right]}. \tag{13}$$

The ith trader's information I_i, is y_i and $P_0^*(y)$. He is able to observe his own sample y_i and $P_0^*(y)$. $P_0^*(y)$ gives the ith trader some information about the sample of other traders. The next section shows that $P_0^*(y)$ reveals "all" the information of the traders.

$P_0^*(y)$ can be interpreted as a stationary point of the following process. Suppose traders initially begin in a naive way, thinking of P_0 as a number and conditioning only on y_i. Let an auctioneer call out prices until the market clears. Call this solution $\bar{P}_0(y)$. That is $\bar{P}_0(y)$ solves

$$\sum_{i=1}^{n} \frac{E\left[\bar{P}_1 | y_i\right] - (1+r)\bar{P}_0(y)}{a_i \operatorname{Var}\left[\bar{P}_1 | y_i\right]} = \bar{X}. \tag{13a}$$

Each period traders come to the market with another realization of $\tilde{y}$, and another $\bar{P}_0(y)$ is found where the auction stops. After many repetitions traders can tabulate the empirical distribution of $(\bar{P}_0, P_1)$ pairs. From this they get a good estimate of the joint distribution of $\bar{P}_0$ and P_1. After this joint distribution is learned, traders will have an incentive to change their bids just as the market is about to clear. This follows from the fact that if everyone observes that the market is about to clear at $\bar{P}_0(y)$, they can condition their beliefs on $\bar{P}_0(y)$ and learn something more about P_1. This changes their demands and thus the market will not clear at $\bar{P}_0(y)$. Suppose instead that the market has been clearing for a long time with prices generated by $P_0^*(y)$, a solution to (12). Then at any particular time, given that traders come to the market with some y, if the market is about to clear at $P_0^*(y)$, and traders then realize that $P_0^*(y)$ is the equilibrium, they will not change their bids due to the new information they get about P_1 from $P_0^*(y)$. $P_0^*(y)$ is a self fulfilling expectations equilibrium: when all traders think prices are generated by $P_0^*(y)$, they will act in such a way that the market clears at $P_0^*(y)$.

3. There is an Equilibrium Price Which is a Sufficient Statistic

Assume that in (4), ϵ_i is a random variable which is normally distributed, with mean 0 and variance 1. Thus, each trader "i" observes $y_i = P_1 + \epsilon_i$, and *given* P_1, y_i is Normal with mean P_1 and variance 1. Each trader gets information of equal precision in that $\operatorname{Var}\epsilon_i = 1$ for each trader "i". Further assume that $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ is jointly normally distributed and covariance $(\epsilon_i, \epsilon_j) = 0$ if $i \neq j$. Thus, we assume that the joint density of y given P_1, say $f(y|P_1)$, is multivariate Normal with mean vector $(P_1, P_1, P_1, \ldots, P_1)$ and covariance matrix which is the identity matrix. P_1 is assumed unknown at time zero, however, traders believe that $\tilde{P}_1$ is distributed independently of $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$, and $\tilde{P}_1$ is Normal $(\bar{P}_1, \sigma^2)$. This marginal distribution of $\tilde{P}_1$ has two interpretations. Under a Bayesian interpretation, next period's price is some fixed number, and people represent their uncertainty about the value of that number with a prior distribution which is Normal $(\bar{P}_1, \sigma^2)$. A non-Bayesian interpretation is that nature draws the true price next period from an urn with distribution Normal $(\bar{P}_1, \sigma^2)$. Nature makes the drawing before period 0. After a particular P_1 is drawn, traders do their research and the ith type trader is able to learn the true value of P_1 to within ϵ_i, where $\tilde{\epsilon}_i$ is distributed as Normal $(0, 1)$. Under either interpretation, the following is true:

THEOREM 1. *Under the above assumption about the joint distribution of* y *and* $\tilde{P}_1$, *if* $P_0^*(y)$ *is given by*

$$P_0^*(y) = \alpha_0 + \alpha_1 \bar{y}, \qquad \textit{where} \tag{14}$$

$$\bar{y} \equiv \sum_{i=1}^{n} \frac{y_i}{n}, \qquad \textit{and} \tag{15}$$

$$\alpha_0 \equiv \frac{\bar{P}_1 \sum_{i=1}^{n} \frac{1}{a_i} - \sigma^2 \bar{X}}{(1+n\sigma^2)(1+r) \sum_{i=1}^{n} \frac{1}{a_i}}, \qquad \textit{and} \tag{16}$$

$$\alpha_1 \equiv \frac{n\sigma^2}{(1+n\sigma^2)(1+r)}, \tag{17}$$

then $P_0^*(y)$ *is an equilibrium. That is, it is a solution to* (12).

Before proving the theorem we present some comments on its significance. First $\bar{y}$ is the sample mean of the y_i. The equilibrium price depends on the information y only through $\bar{y}$. Second, any trader by observing the value of $P_0^*(y)$ can learn $\bar{y}$ from (14), since by (17), $\alpha_1 > 0$. $\bar{y}$ is a more precise estimate of P_1, than is y_i. Thus the market price aggregates all the information collected by the traders in an "optimal" way. $\bar{y}$ is a sufficient statistic for the family of densities $f(y \mid P_1)$. The market aggregation is optimal to the extent that it produces a sufficient statistic.

The following lemma is used to prove the theorem:

LEMMA 1. *If* $h_i(y_i, \bar{y} \mid P_1)$ *is the joint density of* $\bar{y}$ *and* y_i *conditional on* P_1, *then there are functions* $g_1(\cdot)$ *and* $g_2(\cdot)$ *such that, for all* y_i *and* $\bar{y} = \sum_{i=1}^{n} y_i / n$,

$$h_i(y_i, \bar{y} \mid P_1) = g_1(y_i, \bar{y}) g_2(\bar{y}, P_1). \tag{18}$$

That is, $\bar{y}$ *is a sufficient statistic for* $h_i(y_i, \bar{y} \mid P_1)$.

Proof. Conditional on P_1, y_i is Normal $(\bar{P}_1, 1)$ and $\bar{y}$ is Normal $(P_1, 1/n)$. Conditional on P_1, covariance $(y_i, \bar{y}) = 1/n$. Thus conditional on P_1, $(y_i, \bar{y})$ is

$$\text{Normal}\left(\begin{pmatrix} P_1 \\ P_1 \end{pmatrix}, \begin{pmatrix} 1 & 1/n \\ 1/n & 1/n \end{pmatrix} \right).$$

Hence

$$h_i(y_i,\bar{y}\,|\,P_1)=(2\pi)^{-1}\begin{vmatrix} 1 & 1/n \\ 1/n & 1/n \end{vmatrix}^{-1/2}$$

$$\times \exp\left\{-\frac{1}{2}\begin{pmatrix} y_i-P_1 \\ \bar{y}-P_1 \end{pmatrix}'\begin{pmatrix} 1 & 1/n \\ 1/n & 1/n \end{pmatrix}^{-1}\begin{pmatrix} y_i-P_1 \\ \bar{y}-P_1 \end{pmatrix}\right\}$$

$$=(2\pi)^{-1}\frac{n}{\sqrt{n-1}}\exp\left\{-\frac{1}{2}\frac{n}{n-1}\left[(y_i-P_1)^2-(\bar{y}-P_1)(y_i-P_1)\right.\right.$$

$$\left.\left.+(P_1-y_i)(\bar{y}-P_1)+n(\bar{y}-P_1)^2\right]\right\}.$$

Define

$$g_1(y_i,\bar{y})\equiv(2\pi)^{-1}\frac{n}{\sqrt{n-1}}\exp\left\{-\frac{1}{2}\frac{n}{n-1}(y_i^2-2\bar{y}y_i)\right\},$$

$$g_2(\bar{y},P_1)\equiv\exp\left\{-\frac{1}{2}\frac{n}{n-1}\left[2P_1\bar{y}-P_1^2+n(\bar{y}-P_1)^2\right]\right\}. \tag{19}$$

Then $h_i(y_i,\bar{y}\,|\,P_1)=g_1(y_i,\bar{y})g_2(\bar{y},P_1)$. QED

We use Lemma 1 to prove Lemma 2 below. Lemma 2 states that if a trader is given $\bar{y}$, then y_i provides no additional information about P_1 over that provided by $\bar{y}$.

LEMMA 2. *Let $m(P_1|\bar{y})$ be the density of P_1 conditional on $\bar{y}$. Let $\hat{m}(P_1|\bar{y},y_i)$ be the density of $\tilde{P}_1$ conditional on $\tilde{y}$ and y_i. Then $m(P_1|\bar{y})=\hat{m}(P_1|\bar{y},y_i)$ and hence $E[\tilde{P}_1|\bar{y}]=E[\tilde{P}_1|\bar{y},y_i]$, and $Var[\tilde{P}_1|\bar{y}]=\mathrm{Var}[\tilde{P}_1|\bar{y},y_i]$.*

Proof. By Bayes rule,

$$\hat{m}(P_1|\bar{y},y_i)=\frac{g(P_1)h_i(y_i,\bar{y}\,|\,P_1)}{\int_{-\infty}^{\infty}g(P_1)h_i(y_i,\bar{y}\,|\,P_1)dP_1}, \tag{20}$$

where $g(P_1)$ is the marginal density of $\tilde{P}_1$.

$$\hat{m}(P_1|\bar{y},y_i)=\frac{g(P_1)g_1(y_i,\bar{y})g_2(\bar{y},P_1)}{\int_{-\infty}^{\infty}g(P_1)g_1(y_i,\bar{y})g_2(\bar{y},P_1)dP_1}=\frac{g(P_1)g_2(\bar{y},P_1)}{\int_{-\infty}^{\infty}g(P_1)g_2(\bar{y},P_1)dP_1}. \tag{21}$$

The density of $\bar{y}$ given P_1, $f(\bar{y}|P_1)$, satisfies:

$$f(\bar{y}|P_1)=\int_{-\infty}^{\infty} h_i(y_i,\bar{y}|P_1)dy_i=\int_{-\infty}^{\infty} g_1(y_i,\bar{y})g_2(\bar{y},P_1)dy_i$$

$$=g_2(\bar{y},P_1)\int_{-\infty}^{\infty} g_1(y_i,\bar{y})dy_i. \tag{22}$$

The second equality in (22) follows from (18). By Bayes rule

$$m(P_1|\bar{y})=\frac{g(P_1)f(\bar{y}|P_1)}{\int_{-\infty}^{\infty} g(P_1)f(\bar{y}|P_1)dP_1}=\frac{g(P_1)g_2(\bar{y},P_1)\int_{-\infty}^{\infty} g_1(y_i,\bar{y})dy_i}{\int_{-\infty}^{\infty} g(P_1)g_2(\bar{y},P_1)\left[\int_{-\infty}^{\infty} g_1(y_i,\bar{y})dy_i\right]dP_1}, \tag{23}$$

where the second equality follows from (22), $\int_{-\infty}^{\infty} g_1(y_i,\bar{y})dy_i$ can be cancelled from the numerator and denominator in (23), hence

$$m(P_1|\bar{y})=\frac{g(P_1)g_2(\bar{y},P_1)}{\int_{-\infty}^{\infty} g(P_1)g_2(\bar{y},P_1)dP_1}. \tag{24}$$

Comparing (24) and (21), we see that $m(P_1|\bar{y})=\hat{m}(P_1|\bar{y},y_i)$. QED

An immediate consequence of Lemma 2 is that if a trader is given $\bar{y}$ and y_i, then inferences about P_1 will be made independently of y_i. That is y_i is extraneous information if $\bar{y}$ is known.

We now prove the main theorem. The proof uses the fact that if α_0 and $\alpha_1>0$ are known constants, then the conditional distribution of P_1 given $\alpha_0+\alpha_1\bar{y}$, is the same as the conditional distribution of P_1 given $\bar{y}$.

Proof. We show that if $P_0^*(y)\equiv\alpha_0+\alpha_1\bar{y}$, where α_0 and α_1 are given in (16) and (17), then $P_0^*(y)$ satisfies (12) for all y. From Lemma 2, $E[\tilde{P}_1|y_i,\bar{y}]=E[\tilde{P}_1|\bar{y}]$ and $\text{Var}[\tilde{P}_1|y_1,\bar{y}]=\text{Var}[\tilde{P}_1|\bar{y}]$. Under the distribution assumptions given at the beginning of this section it can be shown that the conditional distribution of $\tilde{P}_1$ given $\bar{y}$ is normal with moments given by

$$E\left[\tilde{P}_1|\bar{y}\right]=\frac{\bar{P}_1}{1+n\sigma^2}+\frac{n\sigma^2\bar{y}}{1+n\sigma^2} \tag{25a}$$

$$\text{Var}\left[\tilde{P}_1|\bar{y}\right]=\frac{\sigma^2}{1+n\sigma^2} \tag{25b}$$

(see Degroot [1970], p. 167). That is, the posterior mean of $\tilde{P}_1$ is a weighted average of the prior mean $\bar{P}_1$ and the sample mean $\bar{y}$. Note that $E[\tilde{P}_1|y_i,\alpha_0+\alpha_1\bar{y}] = E[\tilde{P}_1|y_i,\bar{y}] = E[\tilde{P}_1|\bar{y}]$ if $\alpha_1>0$. Hence

$$E\left[\tilde{P}_1|y_i,P_0^*(y)\right]=\frac{\bar{P}_1+n\sigma^2\bar{y}}{1+n\sigma^2}. \tag{26}$$

Similarly,

$$\mathrm{Var}\left[\tilde{P}_1|y_i,P_0^*(y)\right]=\mathrm{Var}\left[\tilde{P}_1|\bar{y}\right]=\frac{\sigma^2}{1+n\sigma^2}. \tag{27}$$

Using (13), (14), (26), and (27)

$$\sum_{i=1}^{n} X_i^d[P_0^*,y_i]=\sum_{i=1}^{n}\left\{\frac{\dfrac{(\bar{P}_1+n\sigma^2\bar{y})}{(1+n\sigma^2)}-(1+r)(\alpha_0+\alpha_1\bar{y})}{\dfrac{a_i\sigma^2}{(1+n\sigma^2)}}\right\}.$$

Using the definitions of α_0 and α_1 given in (16) and (17), (28) becomes

$$\sum_{n=i}^{n} X_i^d[P_0^*,y_i]$$

$$=\sum_{i=1}^{n}\left\{\frac{\dfrac{(\bar{P}_1+n\sigma^2\bar{y})}{(1+n\sigma^2)}-(1+r)\left[\dfrac{\bar{P}_1\sum_{i=1}^{n}\frac{1}{a_i}-\sigma^2\bar{X}}{(1+n\sigma^2)(1+r)\sum_{1}^{n}\frac{1}{a_i}}+\dfrac{n\sigma^2}{(1+n\sigma^2)(1+r)}\cdot\bar{y}\right]}{\dfrac{a_i\sigma^2}{(1+n\sigma^2)}}\right\}. \tag{29}$$

The right hand side of (29) reduces to $\bar{X}$. Thus for all $\bar{y}$

$$\sum_{i=1}^{n} X_i^d[P_0^*,y_i]=\bar{X}. \quad \text{QED}$$

Thus, in equilibrium the current price summarizes all the information in the market. Each trader finds his own y_i redundant. This creates strong disincentives for investment in information, since each trader could do as well by observing only the spot price, as he could if he also purchased a y_i.[1] Note that perfect competition is assumed among the traders, so that the information of *all* type "i" traders taken together affects P_0. However each individual trader of type "i" assumes that his trading activity has no affect on P_0. Thus, when one type "i" trader stops getting information via y_i, P_0 is not affected, and $\bar{y}$ can still be deduced from P_0.

If it costs $C>0$ dollars to become informed then equilibrium will not exist. Each trader of type "i" stops collecting information because the information in P_0^* is superior to y_i and free. Is there another equilibrium with fewer types of informed traders? No. Let there be m types of informed traders, then the price will be a linear function of $\sum_{j=1}^{m} y_j$ and thus transmits all information to uninformed traders. Consider any given informed trader of type m; he feels that he could stop paying C dollars and though he would no longer get the information y_m, the price system reveals the superior information, $\sum_{j=1}^{m} y_j$. Hence, it is not an equilibrium to have m types of informed traders. If no traders are informed, then (for sufficiently small cost of becoming informed) each trader would want to become informed because he gets no information for free via the price system. Hence, with information costs positive equilibrium does not exist. The key to the argument is that no matter how many types of informed traders there are, the price system perfectly aggregates their information and removes the incentive from a trader of a particular type to become informed. This is because traders are price takers and assume the price system is not affected by their actions.

The result that there exists a price which is a sufficient statistic is not robust. For example if the dimensionality of the price system (i.e., the number of commodities less one) is smaller than the dimensionality of the sufficient statistic, then the price function cannot reveal the sufficient statistic. However, Grossman [1975] shows that when prices do not symmetrize people's information, there is a private incentive to open new markets and thus increase the dimensionality of the price system. The rest of this section is devoted to the uniqueness of equilibrium and the notion of "noise." The following section discusses the welfare aspects of equilibrium.

We now show that if there are two equilibria then they must contain different information. If $P_0^*(y)$ and $P_0^{**}(y)$ are equilibria and they contain the same information, then there exists a strictly monotone function $H(\cdot)$ such that $P_0^{**}(y) = H(P_0^*(y))$. Below we show that either $H(\cdot)$ is the identity mapping (i.e., $P_0^{**}(y) \equiv P_0^*(y)$) or one of them is not an equilibrium.

THEOREM 2. *If $P_0^*(y)$ is an equilibrium, and $P_0^{**}(y)=H(P_0^*(y))$, where $H(\cdot)$ is a strictly monotone function which is not the identity mapping, then $P_0^{**}(y)$ is not an equilibrium.*

1. Another paradoxical aspect of markets where prices are sufficient statistics is that each trader's demand function is a function only of the price and not his own information. If all traders ignore their information how does the information get into the price? This point is strongly related to the fact that the demand function in (13) is not an ordinary demand function. It gives the demands of traders *in equilibrium*. In models where the price conveys information, there is no longer the classical separation between demand functions and equilibrium prices. Classically, demand functions can be derived independently of the distribution of equilibrium prices. Here this is no longer possible. See Grossman and Stiglitz [1976] for an elaboration of this point.

Proof. $E[\tilde{P}_1|y_i, P_0^*(y)] = E[\tilde{P}_1|y_i, P_0^{**}(y)]$ and $\text{Var}[\tilde{P}_1|y_i, P_0^*(y)] = \text{Var}[\tilde{P}_1|y_i, P_0^{**}(y)]$ since P_0^* and P_0^{**} contain the same information (i.e., they generate the same σ-field). Let $X_i^d[P_0^*, y_i]$ be as defined in (13). Then if y' is some realization of $\tilde{y}$,

$$\bar{X} = \sum_i X_i^d[P_0^*, y_i'] = \sum_{i=1}^n \frac{E\left[\tilde{P}_1|y_i', P_0^*\right] - (1+r)P_0^*(y')}{a_i \text{Var}\left[\tilde{P}_1|y_i, P_0^*\right]}$$

$$= \sum_{i=1}^n \frac{E\left[\tilde{P}_1|y_i', P_0^{**}\right] - (1+r)P_0^*(y')}{a_i \text{Var}\left[\tilde{P}_1|y_i', P_0^{**}\right]}. \tag{30}$$

Assume, without loss of generality, that $P_0^*(y') > P_0^{**}(y') > 0$. Then

$$\sum_{i=1}^n \frac{E\left[\tilde{P}_1|y_i', P_0^{**}\right] - (1+r)P_0^*(y')}{a_i \text{Var}\left[\tilde{P}_1|y_i', P_0^{**}\right]} < \sum_{i=1}^n \frac{E\left[\tilde{P}_1|y_i', P_0^{**}\right] - (1+r)P_0^{**}(y')}{a_i \text{Var}\left[\tilde{P}_1|y_i', P_0^{**}\right]}. \tag{31}$$

The right hand side of (31) is just $\sum_i X_i^d[P_0^{**}, y_i]$. Thus by (30), $\sum_i X_i^d[P_0^{**}, y_i'] > \bar{X}$. (This clearly also holds in a non-degenerate neighborhood of y_i', as the prices are continuous functions of y.) Thus P_0^{**} is *not* an equilibrium. QED

Thus there cannot be two equilibria with the same information content. The appendix shows that equilibrium is unique in the class of all linear functions of y. We do not know whether there are prices which are non-linear functions of y and are also equilibria.

The result that there is an equilibrium which is a sufficient statistic is not robust. It will not hold if there is noise in the price system. (See Grossman [1975] for an elaboration of the notion of "noise.") Suppose the total stock of the risky asset, x, is unknown to all traders. Suppose that they have a common prior distribution on $\tilde{x}$, such that $\tilde{x}$ is independent of $\tilde{P}_1$ and $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$. When x is random all traders will know that the price which clears the market depends not only on y but also on the realization of $\tilde{x}$. Define an equilibrium as a mapping $P_0(y,x)$, such that for all (y,x)

$$\sum_{i=1}^n \frac{E\left[\tilde{P}_1|y_i, P_0(y,x)\right] - (1+r)P_0(y,x)}{a_i \text{Var}\left[\tilde{P}_1|y_i, P_0(y,x)\right]} = x. \tag{32}$$

Clearly an equilibrium $P_0(y,x)$ cannot be a constant function of x (i.e., a function of y alone). This is because if $P_0(y,x)$ is a function only of y, then the left hand side of (32) *does not* depend on x, while the right hand side of (32) *does* depend on x. As x and y are independent this is impossible. Thus it will not be possible to infer $\bar{y}$ from $P_0(y,x)$ unless x has a degenerate distribution.

4. Welfare Aspects of Equilibrium and Conclusions

Let $\overline{X}_{Fi}$ be the ith trader's endowment of the risky asset. Define

$$u_i^*(X_{Fi},X_i,y) \equiv E\left[U_i(\tilde{W}_{1i})|\bar{y}\right] = E\left[U_i((1+r)X_{Fi} + \tilde{P}_1 X_i)|\bar{y}\right]. \tag{33}$$

Consider the pure exchange economy where traders have utility function $u_i^*(\cdot)$ and endowments are $(\overline{X}_i, \overline{X}_{Fi})$. A competitive equilibrium for such an economy is Pareto efficient. The equilibrium $P_0^*(y)$ is an equilibrium for this economy[2] because it gives each trader the information $\bar{y}$, and this is equivalent to having y. The utility frontier of the central planner with information $\bar{y}$ is equivalent to the utility frontier with information y, since $\bar{y}$ is a sufficient statistic. Therefore the equilibrium $P_0^*(y)$ is efficient to the extent that it yields allocations which a central planner with all the information y would choose as optimal.[3] If there are any other equilibria they cannot yield more efficient allocations.[4] Similarly the noisy equilibrium $P_0(y,x)$ cannot yield more efficient allocations.

The paradox we must face is that $P_0^*(y)$, by being so efficient, removes incentives for individuals to collect information. If information is costly then no individual will purchase it if he can observe P_0^*. Therefore $P_0^*(y)$ is not an equilibrium if information is costly. Only an imperfect information equilibrium can be an equilibrium in an economy where information is costly. There may be imperfect information equilibria. These equilibria would have a chance of persisting in an economy where information is costly.

Hayek ([1945], p. 527) has written:

"We must look at the price system as...a mechanism for communicating information if we want to

2. Where $W_{0i} \equiv \overline{X}_{Fi} + P_0^*(y)\overline{X}_i$.

3. We have shown that for each y the central planner cannot dominate the competitive allocations of Section 3. However, if we consider a replicated economy the variance of y will make life more risky in the competitive economy. A central planner could counteract this by equilizing allocations for a given individual across different realizations of $\tilde{y}$. Thus a central planner could achieve, for all i, higher Eu_i^* than the competitive market of Section 3 even though the planner could not improve $E[u_i^*|y]$ for each individual. This occurs because in Section 3 we have not given the competitive economy the ability to insure against the risks of variation in W_0 due to variation in $\bar{y}$. If before traders observe P_0, we allow traders to trade promises to deliver income contingent on the realizations of $\bar{y}$, then the competitive market will do as well as the central planner in allocating the risks associated with $\bar{y}$. Once the market for risky assets opens the equilibrium price will be a sufficient statistic just as in Section 3.

4. Because of the strong portfolio separation property of the utility functions, the competitive equilibrium holdings of risky assets will be the same when all traders have the same beliefs *irrespective* of which beliefs they have, as long as $\tilde{P}_1$ is conditionally normally distributed. For a given y, the equilibrium allocations generated by P_0^*, strictly Pareto dominate the allocations generated in the naive economy where people observe only y_i and prices are given by $\overline{P}_0(y)$ in (13a). However, the allocations generated by P_0^* do not Pareto dominate the allocations generated by the competitive equilibrium where all traders ignore y_i and use only their prior distribution on $\tilde{P}_1$. This result that no information is as good as all the information is a pecularity of utility functions which have the strong portfolio separation property, and is of little interest. It will always be true that when prices are sufficient statistics the central planner with all information will not be able to Pareto dominate the competitive allocations conditional on y.

understand its real function... The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action...by a kind of symbol, only the most essential information is passed on... ."

In an economy with complete markets, the price system does act in such a way that individuals, observing only prices, and acting in self interest, generate allocations which are efficient. However, such economies need not be stable because prices are revealing so much information that incentives for the collection of information are removed. The price system can be maintained only when it is noisy enough so that traders who collect information can hide that information from other traders. When this occurs some traders want very much to know why prices are, for example, unusually high. It is not enough for traders to observe only prices.

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