How to reconcile Market Efficiency and Technical Analysis

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Abstract

Weak form of the Efficiency Market Hypothesis (EMH) excludes predictions of future market movements from historical data and makes the technical analysis (TA) out of law. However the technical analysis is widely used by traders and speculators who steadely refuse to consider the market as a "fair game" and survive with such believe. In the paper we make a conjecture that TA and EMH correspond to different time regimes and show how both technical analysis predictions for short times and realistic statistical data for larger times can be obtained in a simple single stock model of Gauge Theory of Arbitrage.

Key words: technical analysis, arbitrage, market equilibrium

1 Introduction

For many decades people who deal with securites are divided into two groups. The first group "feels" the market, listens how the market "breathes" and treat the market as alive being [1]. To do this they analyze historical data for prices and volumes, draw patterns and construct indicators, i.e. make use of machinery of the technical analysis (TA) [2, 3]. They are technicians. If somebody asks them whether the price is a random process the answer will be emotional and strongly negative. No one trader would agree that his job is equivalent to throwing a dice. In the same time, the second group, roughly speaking, assumes that the price is a random process, the game is fair and the market is efficient. Indeed, even weak form of the Efficient Market Hypothesis (EMH) sais [7] that the all relevant information came from historical data is encoded in the current price and, hence, the only ingredient

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which is able to influence the future prices is a new information. The information is unpredictable and random. This excludes predictions of future market movements from historical data, i.e. makes the technical analysis out of law. EMH lais in a basis of the financial analysis [7, 8, 9] with many outcomes such as modern portfolio theory [10] and derivative pricing [11].

The conflict lasts for years. Many efforts have been made to check EMH. This leaded to believe that "the evidence for weak-form market efficiency is very strong" and that "technical analysts are deluding themselves about their ability to predict future price movements" [7]. In the same time, there is a number of statistical estimations of TA prediction accuracy (see, for example [14, 15]) which excludes pure random process. Shortly, TA more often is right than wrong but still it is not a "Holy Grail" [16].

Summarizing, traders and speculators mainly stick with technical analysis while fund managers and quantitative analysts base their strategies on EMH. There difference between players suggests that the conflict is due to different time horizons. From this point of view, the TA predictions exists due to an internal deterministic dynamics which brings the market to equilibrium. The market has a long enough memory [17] and the dynamics is not so fast as EMH assumes. We show in the paper that, in fact, TA and EMH can be observed in the same model for a market relaxation and the regime depends only on a time scale. More precise, we show that TA indicators and the corresponding predictions do exist for short times while for large times the model produces EMH state with realistic statistical data.

In a certain sense, the situation with EMH and TA slightly resembles a conflict between Schrödinger and Laplacian determinism in the beginning of the century in physics. Let us remind that the Laplacian (or classical) picture implies the possibility to determine a future state of a dynamical system precisely subject to the condition that the current state is completely known. The Schrödinger's determinism implies that only the probability of the future states of the system can be predicted. The conflict is due to the fact that all we can see around us is deterministic (exactly as extreme apologist of TA may say) but the theory predicts vital influence of quantum fluctuations for long enough times (for example, a car can tunel through a wall if we wait long enough). However, for small enough times any quantum system behaves as classical. Following this line we show in the framework of Gauge Theory of Arbitrage that as time goes the deterministic TA regime is subtituted by EMH regime in course of a relaxation equilibration process.

Recently, in papers [18, 19] the Gauge Theory of Arbitrage (GTA) was introduced to describe relaxation processes in financial market. The approach uses methods of quantum field theory and based on the observation that the discounting procedure can be considered as a parallel transport in a complicated space and curvature of this space is connected with virtual arbitrage opportunities. This allows us to map the theory of capital market onto the theory of quantum system of particles with positive (securities) and negative ("debts") charges which interact with each other through electromagnetic field (gauge field of the arbitrage). In the case of a local virtual mispricing money flows in the region of configuration space (money poor in the profitable security) while "debts" try to escape from the region. Entering positive charges and leaving negative ones screen up the profitable fluctuation and restore the equilibrium in the region where there is no mispricing any more, i.e. speculators washed out the profitable opportunity. It is important to note that in simplest approximation of absence of money flows GTA is equivalent to the assumption about the log-normal walks of assets prices. More realistic statistical characteristics of price series appear if the money flows are taken into account [23]. We discuss GTA in more details in section 4.

The paper is organized as follows. In the next section we main issues of EMH and TA to make the consideration self-contained. In section 3 we give motivation of GTA consideration and introduce the GTA model following [18, 19, 23]. We show how the model leads to TA behaviour in the short time limit and demonstrates a quantitative agreement with market statistical data for large enough times. The paper is concluded with final remarks.

2 Technical Analysis vs Market Efficiency

2.1 Technical Analysis

The technical analysis (TA) in general can be defined as a set of methods for predictions of future prices which is based on“mathematical” rather than economical calculations. It was founded for pure utilitarian purposes, i.e. to get a profit from speculations playing with stocks and, later, with futures. In the same measure as the fundamental analysis is a job for economists, the technical analysis is a field for engineers. It does not mean, however, that there is not place for a fundamental economic information in TA. In paper [15] we showed how to incorporate fundamental informations on a correlation of companies share prices into TA to construct new correlative indicators on the example of two principle energy supply companies in Russia. We believe that the background for a general multiasset consideration is given by Mantegna's ultrametric trees [4] as it was constructed for stocks in DJIA and S& P500 index.

Keeping aside patterns of classical technical analysis, Elliott wave theory [5] and Japanise candlesticks [14] where a recognition of a pattern is quite subjective, it is possible to say that TA consists of a set of simple indicators calculated from previous volumes and prices. The indicators produce “buy” and “sell” signals and can be verified on historical data (this does not implies, however, that the indicators will work in future). It is clear that there is a huge number of combinations of prices and volumes which can be potential indicators but only few of them survive after real data tests.

Mathematically TA is very simple and cannot be compared with complicated financial mathematics involved in portfolio theory, pricing and hedging theory. This causes a certain disbelief in TA. Another source of doubts is a possibility to construct a trading plan, i.e. an algorithm which will substitute a trader. Indeed, the market consists of human beings, reflects human psychology and cannot be put in a finite set of simple equations. Supporters of TA argue that the basic principles of economic theories are not very complicated as well though they developed to describe the same market. Concerning the trading plan, there is an opinion that the plan is useful to save financial and emotional potential of traders in uncertain and fastly changing informational enviroment [6]. In a sense it is analogues to walking in a forest when somebody lost his way – it is better to go straight ahead instead of making loops after loops as a drunk sailor.

Putting short, TA being a contraversial subject, is popular among investors and widely used by professional traders. It is not risk-free. Clear signals do not come very often and it is hard to collect statistics and make money of it. The situation on a market changes with the time and it is important to adjust TA toolkit to it. This fact also smears the statistics. However, TA is more often right than wrong and there are certain market patterns.

Let us now to describe a few TA indicators to illustrate that was said above. First we consider William's % R indicator W_n for n periods (e.g. days) which constructed from previous prices only. It is given by the formula:

$$W_n = -100 \cdot \frac{H_n - C}{H_n - L_n}$$

where C is a last closing price, H_n and L_n are highest and lowest prices for last n days. The indicator oscillates between in the range between -100 and 0. Undervalued zone lays in range -100 and -80(-70), overvalued zone spans from -20 to 0. The "buy" ("sell") signal comes when the indicator leaves the undervalued (overvalued) zone. The method is very simple and is easy to use.

Another example of TA toolkit are Positive Volume Index (PVI) and Negative Volume Index (NVI) [3] which we use in the paper. They are defined by the following rules:

$$PVI_n = PVI_{n-1}(1 + \theta(V_n - V_{n-1}) \cdot r_n) \ , \quad (1)$$

$$NVI_n = NVI_{n-1}(1 + \theta(V_{n-1} - V_n) \cdot r_n) \ , \quad (2)$$

where r_n and V_n are the return and trade volume in n-th period, and $\theta(x)$ is the Heavyside step-function defined as $\theta(x > 0) = 1$ with zero value otherwise. One of interpretations of the PVI assumes that in the periods when volumes increases, the crowd of"uninformed" investors are in the market. These days contribute to the PVI. Conversely, on days with decreasing volume, the "smart money" is quitely taking positions which is reflected by NVI. However, it is important to remember that this is just on of possible interpretations of the indices and they can work because of other reasons. We shall see in section 3 that the"classical" dynamics of the model we analyze reveals predictions of the NVI and PVI indicators though the above interpretation hardly can be used there. "Buy" and "sell" signals appear when the indices cross their own moving averages. Statistical estimations of the indicators accuracy and further references can be found in Ref. [3].

2.2 Efficient Market Hypothesis

In the Introduction we formulated a conflict between TA and Efficient Market. Indeed, if, according to EMH, any relevant information is included in prices already, then there is no way to predict prices from historical data, as TA assumes. We show now that the conflict is apparent and steams from an inaccurate EMH definition.

Let us define Efficient Market following [27] as a superposition of the Rational Expectation Hypothesis and Orthogonality property. *Rational Expectation Hypothesis* states that:

1. Agents are rational, i.e. use any possibility to get more than less if the possibility occur.

2. There exists a perfect pricing model and all market participants know this model.

3. Agents have all relevant information to incorporate into the model.

Using the model and the information the rational agents form an expectation value of the future return $E_t R_{t+1}$. This expectation value can differs from the actual value of the return R_{t+1} on a estimation error $\epsilon_t = R_{t+1} - E_t R_{t+1}$. The *Orthogonality property* implies that:

1. ϵ_{t+1} is a random variable which appears due to comming of new information.

2. ϵ_{t+1} is independent on full information set Ω_t at time t and

$$E_t(\epsilon_{t+1}|\Omega_t) = 0 \ .$$

If agents have a wrong model then the model gives a systematic error and some serial correlation of ϵ_{t+1} and ϵ_τ (with $\tau < t+1$) emerges. For example, if

$$\epsilon_{t+1} = \rho\epsilon_t + \delta_t$$

where ρ is a parameter of the serial correlation and δ_t is a white noise, then:

1. $E_t(\epsilon_{t+1}|\Omega_t) \neq 0$ and

2. $E_t R_{t+1}$ is not a best expectation, i.e. the model is wrong.

Indeed, we can improve the model using $\tilde{E}_t R_{t+1}$ as a new model expectation:

$$\tilde{E}_t R_{t+1} = E_t R_{t+1} + \rho(R_t - E_{t-1} R_{t-1}) \ .$$

If agents do not improve the model in this way they are irrational and there exists a possibility of a superprofit. That is why some tests of the market efficiency concentrate on the existence of the serial correlations and the superprofit. Excelent review of various tests and results can be found in [27]. Almost all of them show that EMH does not hold, at least using the existing pricing models as a candidates for the role of the perfect model.

What is important for our goal here is that the perfect pricing model $E_t R_{t+1}$ is formed using all relevant information and, in particular, historical data. Let us consider an example. If arrival of new information increased a return of a security comparing with other securities with the same measure of risk, then rational traders buy the profitable security and sell others until the returns will not equalize. This equalization (relaxation) process is not infinitely fast, it takes some time and has to be accounted in the perfect pricing model. The knowledge of how the relaxation goes can be obtained from available information and historical data. It means that the analysis of historical data and underlying market forces are extremely important to construct a model of future prices dynamics or, more precise, the dynamics of the expected future prices [30]. At this point we return to the Technical Analysis, which is a set of empirical (phenomenological) rules for expected future prices predictions and the corresponding investment decisions. The comparison of characteristic times of the return fluctuations and the market relaxation defines then the applicability of TA. If we assume that the relaxation time is much smaller than the relaxation one, we return to the simplified EMH definition with all relevant information included in the price and the corresponding price random walk. Following this line we can say that it resolve the conflict between the Technical Analysis and the Market Efficiency.

Summarizing, the Technical Analysis can be considered as a phenomenological method of construction of a mean price model for future prices. It uses price history to estimate the market relaxation and is valid when the relaxation time is not zero. Real prices are stochastically distributed around the mean price and this constitutes the Efficient Market Hypothesis. Now we show how these TA predictions for small times and EMH realistic distribution function for prices can be obtained from the same model. This model is costructed in the framework of Gauge Theory of Arbitrage (GTA) [19], which we describe in the next section.

3 GTA model

When a mispricing appears in a market, market speculators and arbitrageurs rectify the mistake by obtaining a profit from it. In the case of profitable fluctuations they move into profitable assets, leaving comparably less profitable ones. This affects prices in such a way that all assets of similar risk become equally attractive, i.e. the speculators restore the equilibrium. If this process occurs infinitely rapidly, then the market corrects the mispricing instantly and current prices fully reflect all relevant information. However, clearly it is an idealization and does not hold for small enough times [26]. Here, following [19], we give a "microscopic" model to describe the money flows, the equilibration and the corresponding statistical dynamics of prices.

The general picture, sketched above, of the restoration of equilibrium in financial markets resembles screening in electrodynamics. Indeed, in the case of electrodynamics, negative charges move into the region of the positive electric field, positive charges get out of the region and thus screen the field. Comparing this with the financial market we can say that a local virtual arbitrage opportunity with a positive excess return plays a role of the positive electric field, speculators in the long position behave as negative charges, whilst the speculators in the short position behave as positive ones. Movements of positive and negative charges screen out a profitable fluctuation and restore the equilibrium so that there is no arbitrage opportunity any more, i.e. the speculators have eliminated the arbitrage opportunity.

The analogy is apparently superficial, but it is not. It was shown in [19] that the analogy emerges naturally in the framework of the Gauge Theory of Arbitrage (GTA). The theory treats a calculation of net present values and asset buying and selling as a parallel transport of money in some curved space, and interpret the interest rate, exchange rates and prices of asset as proper connection components. This structure is exactly equivalent to the geometrical structure underlying the electrodynamics where the components of the vector-potential are connection components responsible for the parallel transport of the charges. The components of the corresponding curvature tensors are the electromagnetic field in the case of electrodynamics and the excess rate of return in case of GTA. The presence of uncertainty is equivalent to the introduction of noise in the electrodynamics, i.e. quantization of the theory. It allows one to map the theory of the capital market onto the theory of quantized gauge field interacting with matter (money flow) fields. The gauge transformations of the matter field correspond to a change of the par value of the asset units which effect is eliminated by a gauge tuning of the prices and rates. Free quantum gauge field dynamics (in the absence of money flows) is described by a geometrical random walk for the assets prices with the log-normal probability distribution. In general case the consideration maps the capital market onto Quantum Electrodynamics where the price walks are affected by money flows and resemble real trading data.

In simple terms, we consider a composite system of price and money flows. In this model "money" represents high frequency traders with a short characteristic trading time (investment horizon) Δ (for the case of S&P500 below we use 0.5 min as the smallest horizon). The participants trade with each other and investors with longer time horizons. This system is characterized by the joint probability distribution of money allocation and price. If we neglect the money, the price obeys the geometrical random walk which is due to incoming information and longer time horizons traders. The trader's behaviour on time step Δ at price S is described by the decision matrix of non-normalized transition probabilities [19]:

$$\pi(\Delta) = \begin{pmatrix} 1 & t_1 S^{\beta(\Delta)} \\ t_2 S^{-\beta(\Delta)} & 1 \end{pmatrix} \quad (3)$$

where the upper row corresponds to a transition to cash from cash and shares and lower row gives corresponding probabilities for a transition to shares. Parameter t_1 and t_2 represent the transaction costs, bid-ask spread and can model any particular investor's decision making which we want to include in the model. Below we neglect the bid-ask spread and transaction costs but model investor's decision as it shown below. The parameter β is a fitting parameter playing the role of the effective temperature.

At this stage different traders are independent of each other. We introduce an interaction by making hopping elements depending additionally on change in traders configuration. This interaction can model, for example, the "herd" behaviour for large changes and mean-reversion anticipation for small changes. In this case the simplest choice of parameters t_1 and t_2^{-1} is

$$t_2 = t_1^{-1} = e^{\alpha_1(n_1/M-1/2)-\alpha_3\delta(n_1/M-1/2)^3}\ , \quad (4)$$

where n_1 is a number of money amounts (each amount equivalent to the share lot) which have been left in cash for the characteristic time Δ and α_1, α_3 are some numerical constants. It is clear that Eqn(4) describes "herd" behaviour: for large sales (buy off) an investor is also biased to sale (buy), i.e. to follow the "herd". In the same time, small $n_1/M - 1/2$ says that the market is in a stable phase. Then, if some number of traders are selling and lowering the price, an investor considers the situation as a good opportunity to buy anticipating that the price finally will return to its stable value. We will show in section 4 that the simple model (3,4) is able to produce very accurate description of probability distribution function of real prices [23].

Each trader possesses a certain lot of shares or the equivalent cash amount. The formulation of the model is completed by saying that the transition probability for the market is a product of the geometrical random walk weight for price and the matrices (3) for each participant. The total number of participants we assume equal to M.

The matrix $\pi(\Delta)$ has exactly the same form as the hopping matrix for charged particles in Quantum Electrodynamics. This form can also be derived from the assumption that traders want to maximize their profit [19]. In general, it is possible to introduce risk aversion in the model but we do not do it here since it is irrelevant for our purpose. We also do not include many investment time horizons as it was done in [23] to get correct scaling behaviour of the probability distribution function. We return to this point in section 4.

There exist several models which describe pricing in market with many interacting agents. The gauge model has a feature which differs it from earlier ones. The feature is the homogeneity of the traders set. In earlier models traders have always been divided into "smart" (who trade rationally) and "noisy" (who follow a fad) [12, 13]. We believe that for the consideration of short times trades this differentiation is not appropriate. Indeed, all high-frequency market participants are professional traders with years of experience. Unsuccessful traders quickly leave the market and do not affect the dynamics. At the same time, each of the traders has their own view on the market and their own anticipations. That is why their particular decision can be only modeled in a probabilistic way distributing trader's decisions around the rational (true) one. In this sense the traders are neither "smart" (rational) nor "noisy" but a mixture.

Let us state the results for the model. The constructed model allows us to explain quantitatively the observed high-frequency return data. In Ref [29] Figs. 1,2 show the form of the distribution function for changes in the S&P500 market index, which is a price of the portfolio consisting of the main 500 stocks traded on the New York Stock Exchange. The changes in price have been normalized by the standard deviation. In the approximation that the changes are much smaller than the index itself, which is obeyed with very high accuracy, the distribution function of the normalized changes can be considered as the distribution function of the return on the portfolio, normalized by the standard deviation of the return. The return on the portfolio during the period Δ is defined as $r(\Delta) = (S(t+\Delta) - S(t))/S(t)$. In Ref [29] it was also shown that the distribution function obeys the scaling property and that this property is reflected in the dependence on time of the probability to return to the origin. It was demonstrated that for a time period between 1 min and 1000 min (two trading days) the probability decrease as $t^{-\alpha}$ with the exponent $\alpha = 0.712 \pm 0.25$. Similar scaling results have been obtained in Ref. [28] for the high-frequency return for the \$/DM exchange rate with different values of the exponent.

To get the correct scaling behaviour (see Fig 1 in Ref [23]) different time horizons have been introduced to the model with the same dynamical rules. In this part the model follows the Fractional Market Hypothesis (FMH) [25] which states that a stable market consists of traders with different time horizons but with identical dynamics. The "microscopical" electrodynamical model is a model for the dynamics. In the approach the FMH substitutes the information cascade suggested recently to explain the scaling properties of the \$/DM exchange rate [28].

However, to get the realistic profile of the distribution function of returns it is sufficient to use the model we describe above with the only one time horizon. If choose β as $\beta = 30$ and take the number of traded lots infinite, we can plot the probability distribution function of returns for S&P500 as depicted on Fig.2. The same analysis leads to similar results for the \$/DM exchange rate [28] with slightly different values of the parameters. It is easy to see that the theoretical and observed distribution functions coincide exactly with the observed data accuracy. *This demonstrates that the gauge model is able to produce realistic statistical description of real prices in Efficient Market phase.*

Now we turn to the Technical Analysis. It is shown in Appendix that in the limit of small time T the calculation of the transition probability is reduced to the solution of the following "classical" equations which define the dynamics of the system:

$$\begin{aligned} \frac{dy}{dt} &= \sigma^2\Delta\beta^2 M(1/2-\rho) - 2\alpha_1\sqrt{(1-\rho)\rho}\ sinh(v+y) + \left(\frac{d(y+\alpha_1\rho)}{dt} - \sigma^2\beta^2(1/2-\rho)\right)(0)\ , \\ \frac{dv}{dt} &= \left(\sqrt{\rho/(1-\rho)} - \sqrt{(1-\rho)/\rho}\right) cosh(v+y) - 2\sqrt{(1-\rho)\rho}\ sinh(v+y)\ , \\ \frac{d\rho}{dt} &= 2\sqrt{(1-\rho)\rho}\ sinh(v+y)\ . \end{aligned} \tag{5}$$

Here σ is a volatility of the return, $y(t) + \alpha_1\rho = \beta \cdot lnS(t)$ (it gives the return as $\frac{d(y+\alpha_1\rho)}{dt}/\beta$), $v(t)$ is the velocity of the money flows and $1-\rho(t)$ is a relative number of share lots after the last trade. Solutions of the system (5) are presented on Fig.2. We can see that the solutions oscillate with the time. The period of the oscillations depends on the parameters of the model. The return is shifted on a half of the period with respect to $\rho(r)$. Price oscillations fade with time which leads to the market equilibration.

Our goal now is to show that the solutions of the derived equations (5) indeed resemble the situation in the real financial market and are consistent with standard technical analysis tools. The first point here concerns the connection between price movements and trading volumes.

Now when we have ascertained that prices and volumes movements well reflect the real market situation we can estimate the performance of standard TA indicators for our model. The first trivial example is the Price Rate-of-Change (ROC) indicator [3] which is simply the return. So we get "buy" and "sell" signals at points where the security return line crosses some pre-defined levels. One can see that for our idealized description these signals anticipate reversals in the underlying security's price.

Two indicators we described in section 3 can be also easely recognized for our model. They are Positive Volume Index (PVI) and Negative Volume Index (NVI) given by formulae (1,2).

Basing on the previous discussion on the volume behaviour we plot NVI and PVI for our model and estimate the connection between their behaviour and price movements (see Fig.4). First of all, one can see that both PVI and NVI trend in the same direction as prices which agrees with TA arguments. Another important point is that NVI's reversal point in this model always anticipate PVI's ones which is very reasonable if we compare the behaviour of crowd-following and informed investors. Indeed, calculating NVI we take into account only days when the volume describes and the "smart money" is believed to take positions. PVI, on the contrary, counts only days when volume increases, i.e. crowd-following investors are in the market. That is why it seems to be natural that NVI, which reflects "smart money" behaviour, reacts early to changing of the market situation than PVI.

Previous analysis shows that *the gauge model is consistent with TA phenomenological rules of the market relaxation for small enough time.* To answer the question why TA toolkit works also on sufficiently large times we have to return to the model with many time horizons. At characteristic time T the market participants with comparable with T investment horizons are resposible for both the formation of the shape of the distribution function and TA predictions while only all together investment horizons produce correct scaling behaviour.

4 Resume

In the paper we considered relation of the Market Efficiency and applicability of the Technical Analysis. We tried to show that these two issues are not conflicting but complementary to each other. There is a practical outcome of the conclusion. To construct statistical models of the market behaviour it is important to take into account quasideterministic market behaviour on short times. The practical source of information about the behaviour comes from everyday trading practice which is summed up into technical analysis phenomenological rules. These rules can play a role of criterium to construct effective models for the security pricing. It means that to model a particular security it is important to construct the action functional in such a way that the corresponding "classical" equations would be in agreement with most accurate TA tools tested for the security.

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Appendix

In Ref [19] it was shown that the transition probability from the state with the price $S(0)$ and (n_1, m_1) traders in (cash,shares) to the state with the price $S(T)$ and the corresponding traders distribution (n, m) can be written in terms of the functional integral:

$$P(S(T),(n,m)|S(0),(n_1,m_1)) = \frac{1}{n!m!}\int d\psi d\bar{\psi}\ \bar{\psi}_{1,0}^{n_1}\bar{\psi}_{2,0}^{m_1}\psi_{1,N}^{n}\psi_{2,N}^{m}e^{-\bar{\psi}_N\psi_N-\bar{\psi}_0\psi_0}\ I(\bar{\psi},\psi,S(0),S(T))). \tag{6}$$

The functional integral $I(\bar{\psi},\psi,S(0),S(T)))$ has the form:

$$I(\bar{\xi},\xi,S(0),S(T))) = \int DyD\psi_1 D\psi_2 D\bar{\psi}_1 D\bar{\psi}_2 e^{s} \tag{7}$$

with the action

$$s=-\frac{1}{2\sigma^2}\int_0^T \dot{y}^2dt+\int_0^T dt(\frac{d\psi_1^+}{dt}\psi_1+\frac{d\psi_2^+}{dt}\psi_2+\frac{t_1}{\Delta}e^{\beta y}\psi_1^+\psi_2+\frac{t_2}{\Delta}e^{-\beta y}\psi_2^+\psi_1) \qquad (8)$$

and the boundary conditions for integration trajectories:

$$\psi_i(0)=\xi_i\ ,\quad \bar{\psi}_i=\bar{\xi}_i\ ,\quad y(0)=ln(S(0))\ ,\quad y(T)=ln(S(T))\ .$$

The transition amplitudes t_1, t_2 are defined as (4):

$$t_2=t_1^{-1}=exp(\alpha_1(\psi_1^+\psi_1/M-1)-\alpha_3(\psi_1^+\psi_1/M-1)^3)\ .$$

with M is a number of traded lots (both money and shares). In this appendix we derive classical equations of motion (5) for the market on small enough times from the functional integral representation for the transition probability (6,7,8). To this end we first of all consider continuous limit and change y to $\beta y - ln\ t_1$ which results in the following transformation of the action s:

$$s=-\frac{1}{2\sigma^2\beta^2}\int_0^T (\frac{dln\ e^y t_1}{dt})^2dt+\int_0^T dt(\frac{d\psi_1^+}{dt}\psi_1+\frac{d\psi_2^+}{dt}\psi_2+\frac{1}{\Delta}e^{y}\psi_1^+\psi_2+\frac{1}{\Delta}e^{-y}\psi_1^+\psi_1)\ .$$

To extract short time behaviour we now measure time t in terms of smallest time interval in the system, i.e. in units of the time horizon Δ. At such small times (in a stable market) the “herd” effect cannot play any valuable role and we can drop out the term with α_3 in $ln\ t_1$. Since the full number of asset units M is large, we also change fields ψ^+, ψ to the ”hydrodynamical” variables ρ and ϕ which have meaning of density and velocity of the money flows:

$$\psi_i^+=\sqrt{M\rho_i}e^{\phi_i}\ ,\qquad \psi_i=\sqrt{M\rho_i}e^{-\phi_i}\ .$$

The variable $\rho_i(t)$ is proportional to a density of the money flows in the point i at the moment t, while $\phi_1(t)-\phi_2(t)$ gives the corresponding velocity of the flows. In this variables the action takes the form:

$$S(\rho,\phi)=M\int_0^T dt(-\frac{1}{2\sigma^2\Delta\beta^2M}\dot{y+\alpha_1\rho_1}^2+\frac{d\phi_1}{dt}\rho_1+\frac{d\phi_2}{dt}\rho_2+2\sqrt{\rho_1\rho_2}\,cosh(\phi_1-\phi_2+y)) \qquad (9)$$

up to boundary terms which do not contribute to the equations of motion. The functional integral then can be rewritten as

$$I(\bar{\psi},\psi,S(0),S(T)))=\int DyD\phi_1D\phi_2D\rho_1D\rho_2e^{s(\rho,\phi,y)}\ .$$

Appearance of the large external multiplier M is a key point for the calculation of the above functional integral by saddle point method. Indeed, if M tends to infinity the only relevant contribution to the integral are given by the ”classical” trajectories which are defined by the minimization equations:

$$\frac{\delta s(y,\rho,\phi)}{\delta y}=0\ ,\qquad \frac{\delta s(y,\rho,\phi)}{\delta \rho_i}=0\ ,\qquad \frac{\delta s(y,\rho,\phi)}{\delta \phi_i}=0\ .$$

It means that the equations define the joint dynamics of prices and money flows for short enough times. Using explicit form (9) it is easy to check that the last equations can be written as

$$\frac{1}{2\sigma^2\Delta\beta^2 M}\frac{d^2(y+\alpha_1\rho_1)}{dt^2} + \sqrt{\rho_2\rho_1}\; sinh(\phi_1-\phi_2+y) = 0\ , \tag{10}$$

$$-\frac{\alpha_1}{2\sigma^2\Delta\beta^2 M}\frac{d^2(y+\alpha_1\rho_1)}{dt^2}\frac{d\phi_1}{dt} + \sqrt{\rho_2/\rho_1}\; cosh(\phi_1-\phi_2+y) = 0\ ,$$

$$\frac{d\phi_2}{dt} + \sqrt{\rho_1/\rho_2}\; cosh(\phi_1-\phi_2+y) = 0\ ,$$

$$-\frac{d\rho_1}{dt} + 2\sqrt{\rho_2\rho_1}\; sinh(\phi_1-\phi_2+y) = 0\ , \frac{d\rho_2}{dt} + 2\sqrt{\rho_1\rho_2}\; sinh(\phi_1-\phi_2+y) = 0\ . \tag{11}$$

First important note concerns eqn.(10). Indeed, combining eqns (11) and (10) we find the equation

$$\frac{2}{\sigma^2\Delta\beta^2 M}\frac{d^2 y}{dt^2} = \frac{d\rho_2}{dt} - \frac{d\rho_1}{dt} - \frac{2\alpha_1}{\sigma^2\Delta\beta^2 M}\frac{d^2\rho_1}{dt^2}$$

which, after integration, gives us the first order differential equation

$$\frac{dy}{dt} = \frac{M\sigma^2\Delta\beta^2}{2}(\rho_2-\rho_1) + (\frac{d(y+\alpha_1\rho)}{dt} - \frac{M\sigma^2\Delta\beta^2}{2}(\rho_2-\rho_1))(0)\ .$$

Second thing to note is the fact that $\frac{d\rho_1}{dt} + \frac{d\rho_2}{dt} = 0$, i.e. $\rho_1+\rho_2 = const \equiv 1$. This can be checked by taking sum of the eqns(11). It allows us to express ρ_2 as $1-\rho_1$ and finally leads to eqns(5):

$$\frac{dy}{dt} = \sigma^2\Delta\beta^2 M(1/2-\rho) - 2\alpha_1\sqrt{(1-\rho)\rho}\; sinh(v+y) + (\frac{d(y+\alpha_1\rho)}{dt} - \sigma^2\beta^2(1/2-\rho))(0)\ ,$$

$$\frac{dv}{dt} = (\sqrt{\rho/(1-\rho)} - \sqrt{(1-\rho)/\rho})\; cosh(v+y) - 2\sqrt{(1-\rho)\rho}\; sinh(v+y)\ ,$$

$$\frac{d\rho}{dt} = 2\sqrt{(1-\rho)\rho}\; sinh(v+y)\ .$$

Here we introduced the notations $\rho \equiv \rho_1$ for the relative number of "traders" in cash and $v = \phi_2 - \phi_1$ for the velocity of the money flows.

References

[1] R.B.Roosevelt: Metaphors For Trading, Technical Analysis of Stocks & Commodities, **16**, N2 (1998), 15;

[2] T.S. Chande and S. Kroll, *The New Technical Trader*, John Wiley & Sons, 1994;

[3] S.B. Achelis, *Technical Analysis from A to Z*, Irwin Professional Publishing 1995;

[4] T.N. Mantegna, *Hierarchical Structure in Financial Markets*, preprint cond-mat/9802256, 1998; available at http://xxx.lanl.gov/abs/cond-mat/9802256;

[5] R.R. Prechter, Jr. and A.J.Frost, Elliott wave principle key to market behavior, New Classics Library, 1995;

[6] T.Hartle: Opening position, Technical Analysis of Stocks & Commodities, **16**, N3 (1998), 6;

[7] D. Blake, *Financial Market Analysis*, McGraw-Hill, 1990;

[8] S. Lumby, *Investment appraisal and financial decisions*, Chapman & Hall, 1994;

[9] D. Duffie, *Dynamic Asset Pricing Theory*, Princeton University Press, 1992;

[10] E.J. Elton, M.J. Gruber, *Modern portfolio theory and investment analysis*, John Wiley & Sons, 1995;

[11] J.C. Hull, *Options, futures and other derivatives*, Prentice Hall International, Inc, 1997;

[12] J.B. De Long, A. Shleifer, L.H. Summers and R.J. Waldmann: Noise Trader Risk in Financial Markets, *Journal of Political Economy*, **98**, N4, (1990), 703-738;

[13] P. Bak, M. Paczuski, M. Shubik: Price variations in a stock market with many agents, *PHYSICA* A 246, N.3-4, (1997) 430-453;

[14] G.L. Morris, *Candlestick Charting Explained*, Irwin Professional Publishing 1995;

[15] A.Ilinskaia and K.Ilinski, *Russian Securities Market Journal*, **21**, (1997), ??;

[16] M.J. Pring, *Investment Psychology Explained*, John Wiley & Sons, 1993;

[17] A. Arneodo,J.-F. Muzy and D. Sornette: *Causal cascade in the stock market from the "infrared" to the "ultraviolet"*, preprint cond-mat/9708012; available at http://xxx.lanl.gov/abs/cond-mat/9708012;

[18] K. Ilinski: *Gauge Theory of Arbitrage*, Iphys Group working paper IPHYS-1-97, 1997;

[19] K. Ilinski: *Physics of Finance*, preprint hep-th/9709???, available at http://xxx.lanl.gov/abs/cond-mat/9708012; to appear in Edited volume on Econophysics, Kluwer Publishing 1998;

[20] P. Wilmott, S. Howison and J. Dewynne, *The Mathematics of Financial Derivatives*, Cambridge University Press, 1995;

[21] T.Eguchi, P.B. Gilkey and A.J. Hanson: Gravitation, gauge theories and differential geometry, *Physics Reports*, **66** N6, 213 (1980); B.A. Dubrovin, A.T. Fomenko, S.P. Novikov: "Modern Geometry – Methods and Applications", Springer-Verlag, 1984;

[22] M. Creutz, *Quarks, gluons and lattices*, Cambridge University Press, 1983;

[23] K. Ilinski and A. Stepanenko: *Electrodynamical model of quasi-efficient market*, preprint ewp-fin/9805007, 1998; available at http://wueconb.wustl.edu/eprints/fin/papers/9805/9805007.abs

[24] K. Ilinski and G. Kalinin: *Black-Scholes equation from Gauge Theory of Arbitrage*, preprint hep-th/9712034; preprint ewp-fin/9712001; available at http://xxx.lanl.gov/abs/hep-th/9712034;

[25] E.E. Peters, *Fractional Market Analysis*, John Wiley & Sons,Inc., 1995;

[26] G. Sofianos: Index Arbitrage Profitability, NYSE working paper 90-04; The Journal of Derivatives, **1**, N1 (1993);

[27] K. Cuthbertson, *Quantitative financial economics*, Jonh Wiley & Sons, 1996;

[28] S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner and Y. Dodge: Turbulent cascades in foreign-exchange markets, *Nature* **381**, (1996) 767-770;

[29] R.N. Mantegna and H.E. Stanley: Scaling behavior in the dynamics of an economical index, *Nature*, **376**, (1995) 46-49;

[30] Some authors agree that TA shall be accounted in the pricing model but they argue that the purpose of it is to account a presence of "noise" traders who use TA. This scenario also leads to reemerging of TA patterns. This does not contradict to the point of view elaborated in the main text where the patterns are associated with collective market forces since these forces can be caused by the quasi-rational (partially noise) traders whose presence we account in GTA model.

Figure caption

FIG.1 (Ref [23]) Comparison of the $\Delta = 1$ min theoretical (solid line) and observed [29] (squares) probability distribution of the return $P(r)$. The dashed line (long dashes) shows the gaussian distribution with the standard deviation σ equal to the experimental value 0.0508. Values of the return are normalized to σ. The dashed line (short dashes) is the best fitted symmetrical Levy stable distribution [29].

FIG.2 Solution of quasi-classical equations of motion (5). Solid line represents the time dependence of $\beta lnS(t)$, dashed line shows a deviation of the relative number of investors in ρ_1 from its equilibrium value 0.5. Initial conditions are $y(0) = 0.5$, $v(0) = 0.1$ and $\rho(0) = 1/2$. Initial values of first derivatives are equal zero. Time is measured in units of Δ. The parameter α is equal to 0.5 and $M\sigma^2\Delta\beta^2 = 20$.

FIG.3 Prices and volumes from the quasi-classical equations of motion (5). Solid line represents the time dependence of $\beta lnS(t)$, dashed line shows trading volumes as given in the main text.

FIG.4 PVI (long dashes) and NVI (short dashes) constructed from prices and volumes as in Fig.3 Solid line is $\beta lnS(t)$.