

Microscopic Models of Financial Markets

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The Efficient Market Hypothesis vs. the Interacting Agent Hypothesis

EMH: prices *immediately* reflect all forthcoming news about future earning prospects in an *unbiased* manner

-> the statistical characteristics of financial returns are *a mere reflection* of similar characteristics of the news arrival process

Interacting Agent Hypothesis: the dynamics of asset returns arise endogenously from the trading process, market interactions of agents *magnify and transform* exogenous noise (news) into fat tailed returns with clustered volatility.

Inspiration and justification: *results from statistical physics:*

physical systems which consist of a *large number of interacting particles obey universal laws (scaling laws)* that are independent of the microscopic details.

In financial economics:

interacting units -> market participants

scaling laws -> stylized facts: volatility clustering, fat tails

„Statistical physicists have determined that physical systems which consist of a *large number of interacting particles obey universal laws* that are independent of the microscopic details. This progress was mainly due to the development of scaling theory. Since economic systems also consist of a large number of interacting units, it is plausible that scaling theory can be applied to economics“

from: Stanley, H. *et al.* Can Statistical Physics Contribute to the Science of Economics, in: *Fractals* 4 (1996)

Related work:

(1) A variety of interesting work on ‘artificial markets’:

Bak/Paczuski/Shubik, 1996 -> self-organized criticality

Arifovic, JPE 1996 - > GA learning

Santa Fe „artificial stock market“, Arthur *et al.*, 1996

Levy, Levy, Salomon

Stauffer *et al.* -> percolation models

(2) Our approach: statistical models of agent behavior

Theoretical results on dynamics with a large ensemble of agents:

Lux, EJ ‘95 -> herd behavior, bubbles and crashes

Lux, JEDC ‘97, JEBO ‘98 -> chaotic dynamics, theoretical

derivation of variance dynamics

Micro-simulations:

Lux and Marchesi: ‘Scaling and Criticality in a Stochastic Multi-

Agent Model of a Financial Market’, in *Nature*, Jan. 1999.

Chen, Lux, Marchesi: additional features (nonlinearity tests etc.)

Basic Assumptions

(1) different types of traders interact in speculative market:

"*noise traders*" and "*fundamentalists*"

(2) noise traders rely on non-fundamental sources of information:

charts: price trend and

flows: behaviour of others - > mimetic contagion, herding

(3) noise traders are optimistic or pessimistic and reevaluate their expectations in the light of the market's development

(4) traders compare profits gained by noise traders and fundamentalists and *switch to the more successful group*.

(5) traders formulate demand and supply as prescribed by their trading strategy, auctioneer or market maker adjusts the price in the usual

$$\text{manner} \sim \frac{p'(t)}{p} = \beta \cdot ED$$

(6) changes of the (log of the) fundamental value follows a Wiener process: $\ln(p_{f,t}) = \ln(p_{f,t-1}) + \varepsilon_t \Delta t$ with $\varepsilon_t \sim N(0, \sigma_\varepsilon)$

-> *the news arrival process exhibits neither fat tails nor clustered volatility*

Formal representation:

changes of behavior occur according to *state-dependent transition probabilities*:

this means: during a small time increment Δt , one individual will switch between behavioral alternatives (i and j, say) with probability: $\pi_{ij}(t) \Delta t$

In this model:

(1) Switches of noise traders between optimistic and pessimistic subgroup depending on :

majority opinion of other noise traders (*flows*) and

prevailing price trend (*charts*)

transition probabilities:

$$\pi_{+-} = v_1 \exp(U_1) \text{ and } \pi_{-+} = v_1 \exp(-U_1),$$

$$\text{with: } U_1 = \alpha_1 x + (\alpha_2 / v_1) \frac{p'(t)}{p}$$

x: majority opinion (flows), $p'(t)$: price trend

(2) changes between noise trader and fundamentalist group depending on comparison of profits:

actual profits gained by chartists: capital gains (or losses) vs.

expected profits of fundamentalists: percentage difference

between prevailing price and assumed fundamental value

transition probabilities:

$$\pi_{nf} = v_2 \exp(U_2) \text{ and } \pi_{fn} = v_2 \exp(-U_2)$$

with: $U_2 = \alpha_3 * \text{profit differential}$

(3) adjustment of the price [by one elementary unit, e.g. one cent] depending on imbalances between demand and supply.

$$\pi_{\uparrow p} = \max[0, \beta * \text{excess demand}] ,$$

$$\pi_{\downarrow p} = -\min[\beta * \text{excess demand}, 0] .$$

β : reaction speed

Theoretical results

are obtained by analysis of approximate dynamics of first and second moments using the Master equation approach.

Results for the dynamics of mean-values for the price and the number of individuals in each subgroup:

a continuum of a stationary states exists which are characterized by:

(i) price = fundamental equilibrium (on average),

(ii) balanced disposition among noise traders: neither predominance of optimistic nor of pessimistic expectations

(iii) as in equilibrium noise traders and fundamentalists perform equally well: composition of the population is *indeterminate*.

Results for the dynamics of second moments:

autoregressive dependence of (co-)variances plus dependence on mean-values (ARCH effects)

-> ***market appears efficient on average and exhibits***

autocorrelated fluctuations around fundamental equilibrium

Simulations reveal a new phenomenon:

On-off intermittency

Though the system always tends towards a stable equilibrium, it experiences sudden transient *phases of destabilization*.

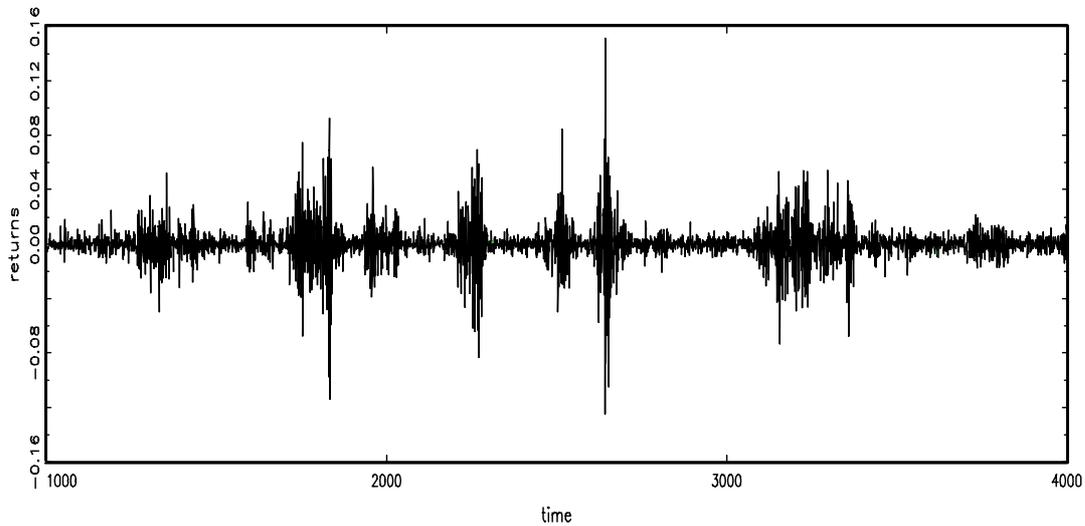
-> the resulting bursts of large oscillations appear as *clustered volatility* in returns.

What happens can be understood as a local bifurcation:

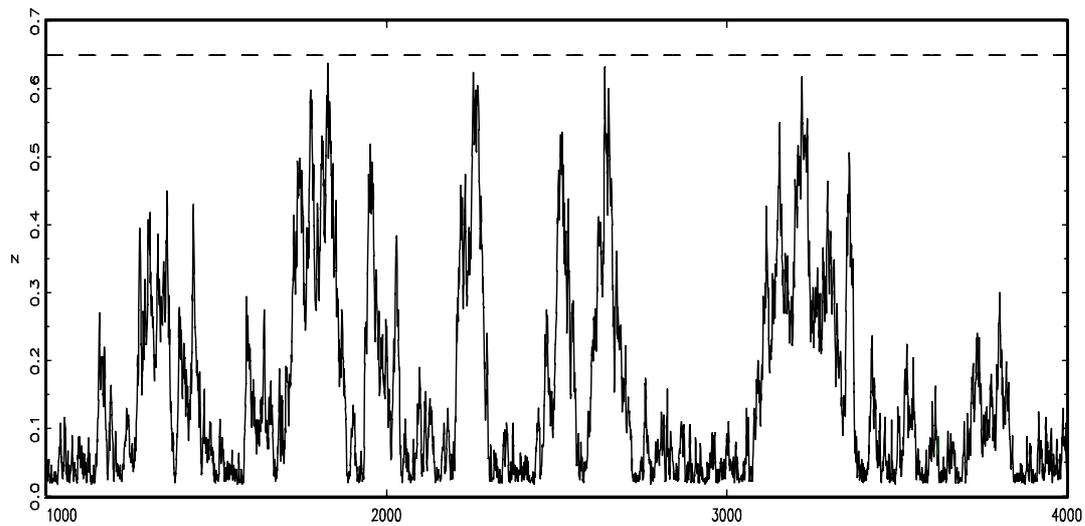
- due to the stochastic nature of the model there is always some noise with *most of the time*: only minor fluctuations around the equilibrium,
- however: *stability* of the equilibrium depends on the fraction of noise traders present,
- every once in a while, stochastic motion or extraneous forces (*news!*) will push the system beyond the stability threshold: onset of severe, but short-lived fluctuations.

-> one observes a mostly stable, but vibrant and fragile market and: the resulting time paths share the basic stylized facts of empirical data.

Simulated time series of returns



Fraction of chartists among traders



Example of the Dynamics: Upper part: typical simulated time series of returns, bottom part: simultaneous development of the fraction of chartists, z . The broken line indicates the critical value $\bar{z} = 0.65$ where a loss of stability is expected given the parameters of the model.

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