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Journal of the American Statistical Association, Volume 62, Issue 318 (Jun., 1967), 321-340.

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Number 318 JUNE, 1967 *Volume 62*

SOME QUANTITATIVE TESTS FOR STOCK PRICE GENERATING MODELS AND TRADING FOLKLORE

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Five stock price sequences are examined quantitatively for structure as predicated by: 1) a random walk model; 2) a continuously differentiable price process; 3) a dynamic model consisting of transients of a discrete process. The first and third models also make predictions in agreement with trading lore. The data are examined by the method of coincident events. Positive evidence is found for both the random walk and discrete transient model, and slightly against the continuous price process. The theoretical predictions seems better confirmed by data at price minima than price maxima. The data are in partial disagreement with the predictions of both the random walk and discrete transient model that large volume and large second differences of price should tend to occur at the same time. Some confirmation is found for items of trading lore not predicted by theory. The non-random properties of stock prices are primarily found in short interval data (daily and weekly) and in individual stock prices as opposed to an average.

1. INTRODUCTION

IN A PREVIOUS paper [7] we described a model for the dynamics of stock trading, which was based on a generalization of the types of orders used to buy and sell stocks and their manner of execution. This model led to a price sequence as a discrete process, being composed of a sequence of finite sections of "starting transients" of a difference equation. These transients were of the form $Ae^{\lambda t}$ ($t=$ an integer) with λ real, complex or imaginary, and were assumed to be started by concentrated bursts of orders, which in turn implies large volume. The conclusions of this model were found to be in qualitative agreement with the "folklore" of stock trading.

It is the purpose of this paper to subject the above qualitative conclusions to a more careful quantitative examination. We shall at the same time test for some properties of other price models, notably the random walk model, and a process which generates prices which are functions of the time, continuous with continuous derivative, and not further specified. We shall also test for the validity of certain concepts in the folklore of stock trading. We shall use the method of coincident events in our analysis, exploiting the analogy between stock market and Geiger counter data [6].

We can motivate our definitions and methods of analysis by some simple examples, which are intended to show that the method of coincident events is merely a formalization of a simple and well known learning process. Let us imagine, in the days when the mercury barometer was first invented, that it was observed once a day, and the height of the mercury or pressure, $p=p\ (t)$, (cf., price of a stock) recorded. Not many days would be required to convince the observer that the measurements fluctuated in the neighborhood of 30 inches, with occasional larger excursions upward or downward. Nor would it be long before an observant individual might suspect that these excursions were associated with unusual weather. Let us formalize this for a very simple example, and indicate some generalizations.

Define a barometric "event" $B\ (t)$ as occuring on any day t on which the recording was less than 28 inches. Suppose there were observed $N_{\text{obs}}(B)=4$ of these in one year, out of $T=365$ "trials". Define a storm event $S(t)$ as a day on which trees were blown down. Suppose $N_{\text{obs}}(S)=5$ of these occurred in our $T=365$ trials.

Now suppose there were two days in the year when a barometric event and a storm occurred on the same day. Let us call this event a simple coincidence or binary event, and denote it by $(B(t),\ S(t))$, the time argument t being the same for both to indicate a coincidence. So we have $N_{\text{obs}}(B(t),\ S(t))=2$. Are these two coincidences convincing evidence that there is a connection between the occurrence of a B and an S event? Intuitively one would certainly be suspicious that such was the case. A calculation will show that the suspicion is certainly justified.

Assume: 1) B events occur independently of other B events; 2) S events occur independently of other S events; 3) B and S events occur independently of each other; 4) B and S events have the same probability from day to day (i.e., time independent). Then the relative frequency, or estimated probability of a B event, for one trial, is $P(B)=N_{\text{obs}}(B)/T=4/365$. For an S event the estimated probability is $P(S)=N_{\text{obs}}\ (S)/T=5/365$. The estimated probability of a coincidence at one trial is the product of these probabilities, $P(B)P(S)$. The "theoretical" or expected number $\mathcal{E}$ of coincidences in T trials is then

$$N_{\text{theor}} = \mathcal{E}(N(B(t),\, S(t)) = 365\cdot(4\cdot 5)/(365)^2 = 0.055$$

The significance probability, or probability of actually observing two or more coincidences under the null hypothesis of the four assumptions above is given approximately by the Poisson distribution. Let $\lambda=0.055$ be the theoretical or expected number of coincidences in a year.

$$P(N_{\text{obs}} \geq 2 \mid \lambda = 0.055) = \sum_{k=2}^{\infty} e^{-\lambda}\lambda^k/k! \cong \lambda^2/2 = 0.0015 \qquad (1)$$

Roughly one year in seven hundred would contain two or more $(B(t),\ S(t))$ coincidences, if these two events occurred independently, and we conclude, at the significance level 0.0015, that barometric and storm events are not independent.

Appendix I contains a derivation of the probability of coincidence when the Poisson formula is not valid. It reduces to the Poisson formula in the case of small numbers of events, relative to the number of trials.

Evidently one can define other types of events and coincidences. A pressure change event would occur if $|\Delta p(t)| = |p(t) - p(t-1)|$ exceeded some specified minimum. A "delayed coincidence" $(|\Delta p(t)|, S(t+1))$, would denote a pressure change event followed in one day by a storm. Colloquially, a negative Δp event might be called a storm predictor (forecasting by one day), whereas the low barometer event B was a storm indicator (happening 'now').

It should be evident from the above discussion that much of folklore or personal knowledge is acquired and expressed by observation of coincident or delayed coincidence events. We are going to define certain events in the sequence in time of stock market prices and volumes, and examine both market folklore and the predictions of some price models, using the method of coincident events. Before doing so we must point out some essential elements in the process of formalizing the method, and some limitations.

First, the data must be expressed as a discrete sequence of trials, even though (as in the case of the barometer) it may be originally expressed continuously. Second, the events must be defined so that just one event of a given type unambiguously did or did not occur just once in each trial. Third, the mathematics is much more simplified, if the events are defined so as to be relatively infrequent compared to the number of trials—i.e., so that the estimated probability (number of events observed (single or coincidences)/number of trials) is small. How small is "small enough" is sometimes debatable. A rule of thumb for the probability would suggest 0.10, or perhaps $2/\sqrt{\text{number of trials}}$, will usually be satisfactorily small. This rule of thumb is suggested by examining the neglected terms and approximations required to obtain equation A6 in the appendix.

We should point out that the Poisson series can be used to test both for an excess or a deficiency, relative to independence of events. In the case of a deficiency, the expected number λ of coincidences must be greater than about 4, or the test for deficiency cannot be very effective. The reason for this is that the "low frequency tail" of the Poisson distribution (Eq. (1)) starts at $k=0$ (no observed binary events at all) and $P(k=0)=e^{-\lambda}$, or $e^{-4}=0.018$. No observed coincidences, if 4 are expected, is not quite significant at the 1% level, but is at the 5% level. This problem can be alleviated in practice by combining observations (see Section 7), using the fact that the sum of Poisson variables is a Poisson variable, to reach a significant conclusion.

While we have indicated the convenience for the events to be infrequent relative to the number of trials, they must not be so infrequent that there is doubt, or insufficient evidence, that probability concepts are applicable. Thus a person may (erroneously) have a lifelong phobia because of one unfortunate coincidence between two events in childhood, one of which was unpleasant. Adults are sometimes impressed by the coincidence of rare events such as some remarkable astronomical phenomenon, and the occurrence of an historical event on earth. But unless in practice both types are unambiguously definable, and recognizable as possibly occurring more than once, there is not much point in using statistics to argue for a connection in the probability sense between them.

Finally we should point out that although the low barometer event was connected with the storm event in the above example, as a storm indicator it was wrong more than half the time. One should not look for infallibility even when the statistical evidence of a connection is quite good. In a practical sense there are many other indicators of a storm available to the observer in addition to the barometer. In our language, he can use triple and higher order coincidences, either as a storm indicator, or a storm predictor. One can think of many analogies to the above statements in the everday, unconscious use of the method of coincident events.

2. EVENTS IN STOCK MARKET SEQUENCES

Let us now turn to stock market data, define some events in an unambiguous way, and indicate the kind of coincidences that might be expected to occur between them on the basis of folklore, or some price-generating models.

The four basic sequences we shall consider, specifying a basic or single "trial" interval of a day, week, or month, are the sequence of high prices $p_h(t)$, low prices $p_l(t)$, closing prices $p_c(t)$, and the volume $V(t)$. These are the four basic sequences plotted on the ordinary chart of stock prices. Note that by definition the closing price must fall in the interval between the high and low price for the same interval, $p_l(t) \leq p_c(t) \leq p_h(t)$. Hence it follows from definitions that some events defined for one of these three sequences will not be independent of other events defined for the other two.

The events below are defined in such a way as to be easily and unambiguously recognized on the charts. They are certainly not the only definable events, and they may not even be the best definitions for the purposes intended. They have only the merit of simplicity and ease of recognition. Fig. 1 illustrates the events and the coincidences defined below.

A simple maximum (minimum) occurs in a sequence when a member of the sequence is greater (less) than the nearest preceding and following member which is different. Note that price and volume data appear in multiples of $\frac{1}{8}$ dollar or one round lot, so that this definition allows several maxima to appear in sequence without an intervening minimum, and conversely.

A ten per cent S (Superior) event is a simple maximum in the sequence of highs $p_h(t)$ for which there were preceding and following trades in the market for the particular stock considered at ten per cent less than the price at S and none greater than at S of closer proximity. A ten per cent I (Inferior) event is similarly defined from minima in the sequence of lows $p_l(t)$. The ten per cent figure is arbitrary; it is simply picked large enough to make the I and S events sufficiently infrequent. I and S events obviously do not occur independently of each other. They tend to avoid each other, and to alternate in appearance for a given stock or average.

An alternative definition of particular S and I events, are the "day away" events S_{da}; I_{da} (reference 3, p. 334). This means, for S_{da}, that there are nearby preceding and following days of trading for which the high-low range is "away from" a closer day's trading; i.e., the high of one day is less than the low of a preceding day's trading nearer to or on the S_{da} day, and similarly for some day preceding the S_{da}. If two S_{da} events occur without an intervening I_{da}, choose only the highest of these S_{da}. If two or more such S_{da} events are highest (ex-

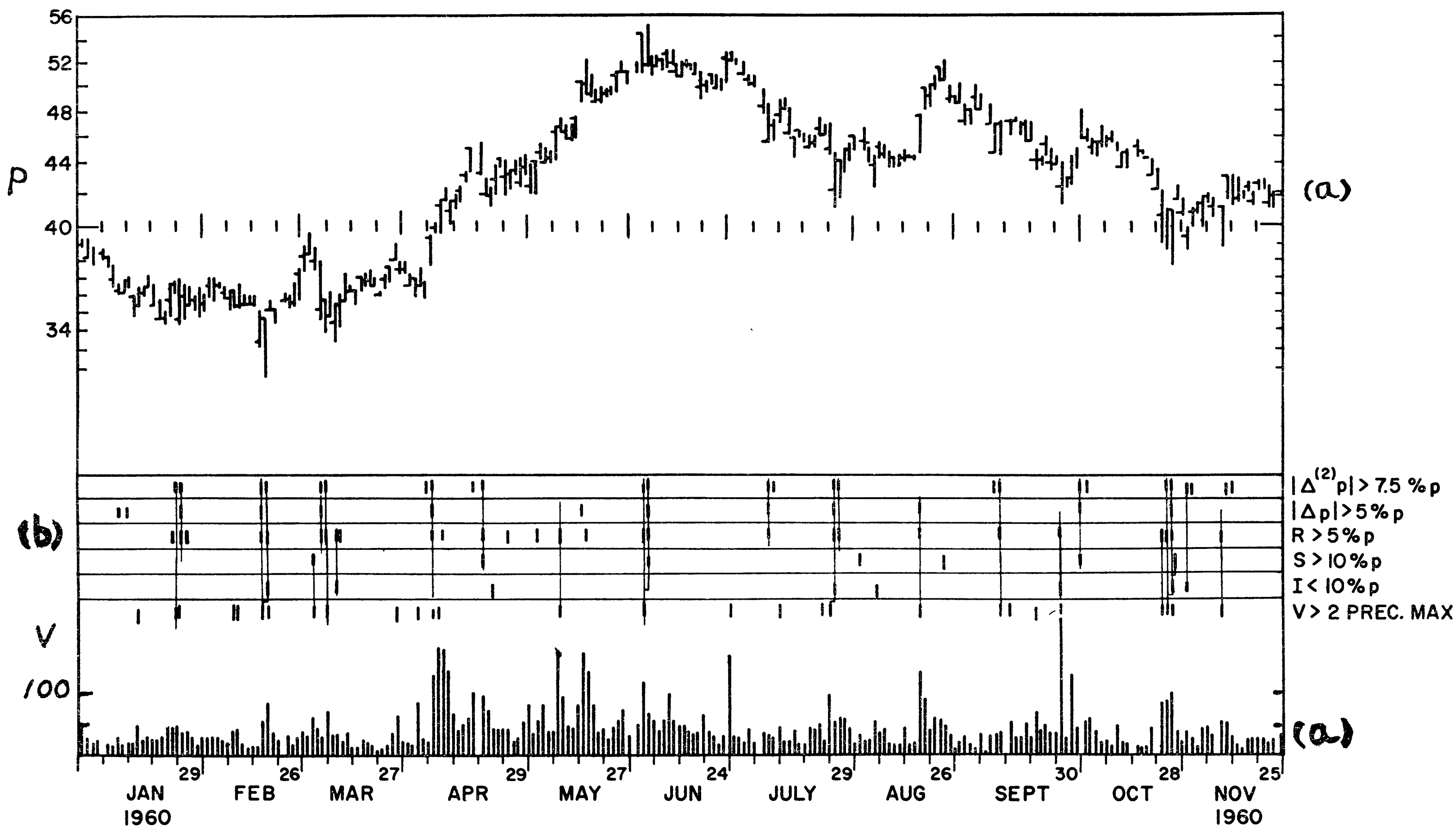

FIG. 1. (a) Daily highs (p_h), lows (p_l), and closing prices p_c, in $ and volumes in 100 share lots of Magnavox Co. stock for December 28, 1959 to November 25, 1960; (b) coincidence chart for price and volume events. The events are labeled and briefly defined at right of coincidence chart. Light line denotes coincidence, stepped light lines denote delayed coincidence. Data taken graphically from charts of Traders Research, Inc., Lambert Airport, Missouri.

actly equal), choose all. S_{da}, I_{da} events are even easier to pick out on charts than per cent S and I events, but occasionally give rise to ambiguities. For example, on Fig. 1 the price maximum in the third week of May never becomes a "day away" maximum. The maximum on June 1 is not identifiable as a "day away" maximum until about June 17.

It is evident that S and I events of either the percentage or day away type cannot be immediately identified as such at the end of the day on which they occurred, but are almost always unambiguously dated for a given span of data.

A volume event is defined as occurring when the volume of any day (week, month) is larger than the two preceding simple maxima in the volume series. From this definition, several volume events can occur in immediate sequence, the last of which will itself be a simple maximum. Generally speaking a volume event will be a day of relatively large volume. But if there is one volume event of quite large volume, we can have quite large volumes on succeeding days without their being defined as volume events. A volume event can be immediately identified at the end of the day (week, month) of its occurrence.

A $|\Delta p|$ event occurs if the change of the *closing* price, in absolute value, $|\Delta p_c(t)| \equiv |p_c(t) - p_c(t-1)|$, is greater in absolute value than some pre-assigned minimum, expressed either in dollars or as a percentage. A $|\Delta p|$ event is (arbitrarily) dated as of the second day (week, month) of the two closing prices needed to generate it; i.e., a $|\Delta p|$ event is considered to occur on a single day. One can also take account of the sign of Δp and refer to $+\Delta p$ or $-\Delta p$ events.

A $|\Delta^{(2)}p|$ event occurs when the absolute value of the second difference of *closing* price, $|\Delta p_c(t) - \Delta p_c(t-1)|$, exceeds some arbitrary minimum. It is considered to "occur" on the last *two* days (weeks, months) of the three consecutive prices needed to generate a single second difference. We use this double dating convention since we interpret a $|\Delta^{(2)}p|$ event as an indication of large (sequential) dispersion [6] in the sequence of numbers $\Delta p(t)$, $\Delta p(t+1)$, etc. $|\Delta^{(2)}p|$ is the range of a sample of two consecutive Δp's from the population of sequential Δp's, and a sample of two is the smallest possible sample which can be used to estimate dispersion.

A range event R occurs when the difference of the two sequences, $p_h(t) - p_l(t)$, on a single day (week, month) is greater than some pre-assigned minimum.

The above six events are defined so that not more than one event of a given kind occurs* in one basic interval, for one sequence. For any one basic interval (day, week, month) we can from the data decide unambiguously that an event of a given type did or did not occur.

It should be noted that of the six events defined above, four of them, V, $|\Delta p|$, $|\Delta^{(2)}p|$, R can be immediately and unambiguously identified as having occurred at the end of any given basic interval. Unfortunately this is not true of the I and S events, though they can be unambiguously identified from a complete sequence of data. I and S events are obviously good points to buy and sell. Hence, if one can find a statistically significant degree of coincidence between I and S events and the remaining four, these four events may provide

* This is not strictly correct for $|\Delta^{(2)}p|$ events, since if four or more large Δp events occurred in sequence, of alternating sign, there would, by our definition be two $|\Delta^{(2)}p|$ events on one day. In practice we have not seen this happen, although theoretically it could occur.

some practical trading signals. In the stock market this is called "technical analysis".

A second point to observe is that the six types of events can be divided into three groups, and within each group the events are certainly not independent of each other. These three groups are:

1. I and S events. These two events tend to avoid coincidence both with themselves and each other, simply from the way they are defined, Thus coincidences of the Type $(I(t), S(t))$ and delayed coincidences of the Type $(I(t-1), I(t))$, occur less often than if these individual events occurred completely independently.

2. Volume events V. Volume events do not appear independently of each other. In fact they tend to cluster, since it is known that volume tends to appear in bursts sometimes lasting longer than a month [6]. Delayed coincidences of the type $(V(t-1), V(t))$ probably appear more frequently than if V events occurred independently of each other.

3. $|\Delta p|$, $|\Delta^{(2)}p|$ and R events also tend to coincide with each other, simply from the way they are defined. The first two are defined from the closing sequence $p_c(t)$, the last from the difference $p_h(t)-p_l(t)$. The high, low and closing sequences are connected by the relation $p_l(t) \leq p_c(t) \leq p_h(t)$. Hence, large first or second differences events in $p_c(t)$; $|\Delta p|$, $|\Delta^{(2)}p|$, and the range event R will all tend to coincide. Binary coincidences of the type $(|\Delta p(t), |\Delta^{(2)}p(t)|)$, $(R(t), |\Delta p(t)|)$, $(R(t), |\Delta^{(2)}p(t)|)$ will all tend to occur more frequently than if the separate events occurred independently (see upper part of Table 1).

What can we say about binary coincidences where the two members are drawn from different groups above, say an $(R(t), V(t))$ or an $(S(t), V(t))$ coincidence? Our definitions do not give us any clues here. One might then reasonably assume independence as a null hypothesis, and then use data to see if the null hypothesis is rejected. Trading folklore, and different price generating models do make definite inferences about coincidences of events from these three different groups, and it is just these inferences we wish to test. In sections 3 through 6 we identify the coincidences to be expected from folklore and price models, with brief references to the tables where data on these coincidences appear. In section 7 we examine this data in detail.

3. COINCIDENCES EXPECTED FROM FOLKLORE

Consider the statement "It takes volume to make prices move". We can translate this into a statement about coincidences between our events as follows: "Move" could mean large motion up or down in one day (week, month), a $|\Delta p|$ event. "Move" could also mean a violent *change* of motion on two successive days, a $|\Delta^{(2)}p|$ event, or violently up and down within a single day, which would mean an R event. So the statement means there are more coincidences of the type

$$(V(t), |\Delta p(t)|), \quad (V(t), |\Delta^{(2)}p(t)|), \quad (V(t), R(t))$$

than are given by the hypotheses that V events occur independently of $|\Delta p|$, $|\Delta^{(2)}p|$ or R events (see lower half of Table I).

A slightly different statement, "It takes volume to put prices up, but they can fall of their own weight", can be translated to imply more coincidences of the type $(V(t), +\Delta p(t))$ than $(V(t), -\Delta p(t))$, the former being more frequent than if V and $+\Delta p$ events occurred independently. We have not tested for this item of folklore.

The statement "important moves tend to end on large volume, or climactically" can be translated into statements about coincidences as follows. The end of an important move is likely to be an I or S events, if the subsequent price motion is in a direction opposite to the previous motion. It does not have to be an I or S event if the "subsequent motion" is in the same direction at a reduced rate, or in fact no motion at all, and the price sits still (in technical analysis a horizontal "line"). "Large volume" evidently implies a volume event. "Climactically" could imply violent motion up and down in one or more successive days, i.e., $|\Delta p|$, $|\Delta^{(2)}p|$, or R events. Taken as a whole this statement suggests looking for more frequent occurrence of coincidences of I and S events (Group 1) with the remaining four (Groups 2 and 3) than would be given by the null hypothesis that I and S events occurred independently of the remaining four events (see Table 2).

In our previous paper [1] we argued that "something happening" in the market implied V, $|\Delta p|$, $|\Delta^{(2)}p|$, and R events. If "something happening" also means an important turning point, or change of sign of the expected value of Δp, this means an I or S event. This is the same as the conclusion just reached.

Some of the more sophisticated tenets of stock data interpretation seem to imply that the volume sequence foretells or leads the price sequence [1]. This statement suggests looking for an excess of delayed coincidences of the form $(V(t-1), S(t))$ and $(V(t-1), I(t))$, and a deficiency in delayed coincidences of the form $(S(t-1), V(t))$, and $(I(t-1), V(t))$, Table 4. The evidence is small but positive. It may be that different definitions of events and coincidences would make better tests for these folklore statements.

4. COINCIDENCES EXPECTED FROM THE RANDOM WALK MODEL

Let us now translate some properties of different price models into statements about coincidences. Suppose the sequence of transacted prices in a given interval, say one day, is a simple coin tossing random walk, with the number of tosses, or steps proportional to the number of transactions, or volume. We further suppose that the random walk on each day starts where it ended at the close of the previous day. From this price model it follows that: (1) $|\Delta p_c(t)|$, the net distance traversed in the walk from its daily starting point, (the previous close); (2) $|\Delta^{(2)}p_c(t)$, a crude measure of the dispersion of the daily sequence of Δp_c's; and (3) the daily range $R(t)$, all increase with the number of steps, or volume. In terms of coincidences, this means an increased frequency of coincidences of the type $(V(t), R(t))$, $(V(t), |\Delta p(t)|$, and $(V(t), |\Delta^{(2)}p(t)|)$, relative to their frequency under the null hypotheses that V events occur independently of R, $|\Delta p|$, and $|\Delta^{(2)}p(t)|$ events. In other words, groups 2 and 3 are not independent, for the random walk model. Note that this prediction of the random walk model is qualitatively the same as what we inferred from the folklore statement that "it takes volume to make prices move". Whether there is a quantitative difference, we do not know, though we suspect there is (Section 7).

The above discussion refers to the relation of big "jumps" ($|\Delta(p)|$, $\Delta^{(2)}p$ events) to volume, for the random walk model. In contrast, the random walk model, so far as we can see, does *not* make any predictions as to where maxima and minima (I and S events) occur; relative to the volume or to $|\Delta p|$ or $|\Delta^{(2)}p|$. I and S events are defined to be of a magnitude such as 10% of the price, and hence are relatively infrequent. We surmise using the following heuristic argument that for this model I and S events occur independently of $|\Delta p|$, $|\Delta^{(2)}p|$, and R events. For a random walk model, the probability of any given combination of single steps (including those which give I or S events) is a constant, independent of what has gone before. If the number of steps between observations, or volume per day varies, the probability distribution of $\Delta p_c(t)$ does vary (notably the dispersion) from day to day, but the mean of the distribution does not. Hence we suspect, but cannot prove, that the appearance by chance of an I or S event in the total sequence is independent of the occurence of V, $|\Delta p|$, $|\Delta^{(2)}p|$ or R events, if the prices are strictly generated by a random walk. In other words, we conclude group 1 events are independent of both groups 2 and 3 events, in the random walk model.

5. COINCIDENCES EXPECTED FROM A CONTINUOUS AND CONTINUOUS DERIVATIVE PRICE SEQUENCE

Let us now suppose a second model, specified as follows. There exists an unknown mechanism which generates prices which are, *for all times, continuously differentiable functions of that time.* The observed, executed transaction prices are the prices generated by this mechanism, sampled at times corresponding to the executions plus an error term, and rounded off to the nearest 1/8. The key words are in italics. The actual mechanism is unimportant.

We can illustrate the force of these remarks by an example. Imagine a weight hung on a spring, and set in vertical oscillation. The height of this weight $p(t)$ (cf. price) is for all times after the start a continuous with continuous derivative function of the time. For such functions the calculus gives as a necessary condition that the derivative dp/dt is zero at maxima and minima. This suggests that for *observations* of $p(t)$ at *discrete* intervals, with an error, the difference, $\Delta p_c(t) \equiv p_c(t) - p_c(t-1)$, and the range $R \equiv p_h(t) - p_l(t)$ should be relatively small at maxima and minima, provided the interval of differencing (i.e. between observations), a day, week or month, is *small* compared to the natural dynamical periods (real or complex) of the mechanism. In terms of our $|\Delta p|$ and R *events*, implying *large* $|\Delta p_c|$ and R, these events should *avoid* coincidence with I and S events. This conclusion is precisely the opposite of what we concluded from the statements of folklore.

Concerning second differences of the "observed" prices, we can make no such general statements. For the particular case of the single mass on the spring, or simple harmonic motion, the absolute value of the second difference of $p(t)$, $|\Delta^{(2)}p(t)|$ is in fact largest at the maxima and minima, but this is not true of all physical systems in general.

In summary therefore, for a price model in which the underlying mechanism gives prices which are at all times continuous with continuous derivative functions of the time, with dynamical periods or "proper values" longer than the interval of observation (day, week or month), I and S events will avoid coincidence with $|\Delta p|$ and R events. There is no indication, from our arguments as to where $|\Delta^{(2)}p|$ and volume events will occur, so for this model we assume these occur independently of I and S.

It should also be noted that the continuous and analytic model assumes observations at intervals (day, week, month) *smaller* than the natural dynamical "periods" of the underlying mechanism. If these day, week or month intervals are *larger* rather than smaller, one could well have the observations fitting a random walk model. "Large", rather than "small" is precisely the condition under which Einstein originally derived the properties of Brownian motion from the *continuous* motion of small dust particles in collision with molecules.

6. COINCIDENCES EXPECTED FROM THE DISCRETE TRANSIENT MODEL

The third model is the one we have already proposed [7]. The price sequence is a set of transients of the form $Ae^{\lambda t}$ for t taking on discrete values only, the transients being initiated by large volume. For this model, as we indicated in our previous paper (see Fig. 3, ref. 7), we expect more coincidences, relative to the hypotheses of independence, of I and S events with $|\Delta p|$, $|\Delta^{(2)}p|$, R, V, i.e., just the coincidences implied by the folklore statement concerning the end of "important moves" (Section 3 and Table 2). This third model also implies excess binary coincidences among V, $|\Delta p|$, $|\Delta^{(2)}p|$ and R. In this respect its predictions are qualitatively the same as for the random walk model, and also with the folklore statement that it takes volume to make prices move (Section 7 and lower half of Table 1).

The above three models do not of course exhaust the possibilities. In particular it should be noted that if the specifications on the continuous model were relaxed from "at all times", to "at all times except a discrete number of points," and these discontinuities of function or derivative were marked by large volume, the resulting conclusions could be quite similar to those which we reached for our discrete transient model. The discrete transient model was built around the concept of a totally discontinuous process. The prices did not exist except at discrete instants of time [7].

7. COMPARISON OF THEORY AND OBSERVATION

Let us now examine some data to see in what respects the conclusions from theory we have drawn in the preceding discussion are supported by the evidence. The five sequences considered, and the criteria defining the events, are given at the top of Table 1. They were selected as follows: the first four sequences were picked out of the supply of semi-log charts (linear charts for Daily *DJI*), available to the author, primarily so as to have an appreciable number of I and S events on them. The existence of sufficient I and S events was needed to test adequately for their coincidence or avoidance of the other events. Stocks which traded in low volume and price were also avoided. Beyond these two specifications no attempt was made to pick favorable or unfavorable cases. This selection introduces some bias into our sample, since this choice means we have picked sequences, or time intervals, in which the stocks really "moved".

TABLE 1. OBSERVED FREQUENCIES OF SINGLE AND BINARY EVENTS. DATA TAKEN GRAPHICALLY FROM CHARTS OF TRADERS RES. INC., AND SEC. RES. CORP.

	Single Events									
	(1)		(2)		(3)		(4)		(5)	
	12/28/59 to 11/25/60 Magnavox		1/2/61 to 12/1/61 Dow-Jones Industrial Total market volume		7/55 to 12/59 United Airlines		7/45 to 3/62 Johns-Manville		7/55 to 6/64 D J Industrial D J vol. only	
	Daily	Criterion	Daily	Criterion	Weekly	Criterion	Monthly	Criterion	Weekly	Criterion
No. of intervals	232		232		233		201		443	
No. of $\lvert\Delta^{(2)}p\rvert$	26	> 7.5% price	28	> 10 points	23	> 10% price	13	> 20% price	22	> 5% price
No. of $\lvert\Delta p\rvert$ events	11	> 5% price	19	> 7 points	13	> 7% price	8	> 12.5% price	12	> 4% price
No. of R events	28	> 5% price	9	> 13 points	21	> 10% price	15	> 15% price	22	> 5% price
No. of S events	7	> 10% price	9	Day away	13	Week away	8	> 20% price	9	> 10% price
No. of I events	8	> 10% price	9	Day away	14	Week away	9	> 25% price	9	> 10% price
No. of V events	28		27		30		30		55	

Binary coincidences expected from definitions of events

	(1)		(2)		(3)		(4)		(5)		Sum of events for coincidence type	
	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}
No. of ($\lvert\Delta p\rvert$, $\lvert\Delta^{(2)}p\rvert$) events	9*	1.2	6*	2.3	6*	1.3	3*	0.5	6*	0.6	30*	5.9
No. of ($\lvert\Delta p\rvert$, R) events	9*	1.3	3*	0.73	7*	1.2	5*	0.6	8*	0.6	32*	4.4
No. of ($\lvert\Delta^{(2)}p\rvert$, R) events	10*	3.1	2	1.1	9*	2.1	3	1.0	10*	1.1	34*	8.4
Sum of events for a sequence	28*	5.6	11*	4.1	22*	4.6	11*	2.1	24*	2.3	96*	18.7

Binary events expected from random walk model, discrete transient model, and folklore ("volume moves prices")

	(1)		(2)		(3)		(4)		(5)		Sum of events for coincidence type		
	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}	N_{obs}	N_{theor}	P
No. of ($\lvert\Delta p\rvert$, V) events	4*	1.3	5	2.2	6*	1.7	2	1.1	4*	1.5	21*	7.8	<0.01
No. of ($\lvert\Delta^{(2)}p\rvert$, V) events	8*	3.1	2	3.2	4	3.0	1	1.9	4	2.7	19	13.9	0.09
No. of (R, V) events	12*	3.4	1	1.04	9*	2.7	6*	2.3	11*	2.7	39*	12.1	<0.01
Sum of events for a sequence	24*	7.8	8	6.4	19*	7.4	9	5.3	19*	6.9	79*	33.8	<0.01

* Denotes $P(n \geq N_{obs}) < 0.05$.

TABLE 2. COINCIDENCES EXPECTED ON BASIS OF DISCRETE TRANSIENT MODEL AND TRADING LORE ONLY ("IMPORTANT MOVES AND CLIMACTICALLY"). OBSERVED AND CALCULATED NUMBER OF COINCIDENCES INVOLVING MAXIMA (S) AND MINIMA (I) EVENTS, USING THE OBSERVED FREQUENCIES OF TABLE I.

$P = P(n \geq N_{obs}) = \sum_{n=N_{obs}}^{\infty} e^{-\lambda} \lambda^n/n!$ where $\lambda = N_{theor}$.

	Binary Coincidences															Sum of events for a coincidence type		
	(1)			(2)			(3)			(4)			(5)					
	N_{obs}	N_{theor}	P	N_{obs}	N_{theor}	P	N_{obs}	N_{theor}	P	N_{obs}	N_{theor}	P	N_{obs}	N_{theor}	P	N_{obs}	N_{theor}	P
Coincidence Event time t is same for both events																		
†(S, $\lvert\Delta p\rvert$)	2	0.33	0.04*	0	0.73	1.0	0	0.73	1.0	0	0.32	1.0	1	0.24	0.17	3	2.3	0.3
(S, $\lvert\Delta^2 p\rvert$)	3	0.77	0.05*	1	1.09	0.66	2	1.3	0.37	0	0.52	1.0	0	0.45	1.0	6	4.13	0.2
†(S, R)	2	0.84	0.21	1	0.33	0.28	0	1.12	1.0	1	0.6	0.7	1	0.45	0.33	5	3.34	0.2
(S, V)	1	0.84	0.57	2	1.04	0.31	1	1.7	0.82	1	1.2	0.7	3	1.1	0.10	8	5.88	0.2
Sum of events for a sequence	8	2.78	0.01*	4	3.19	0.37	3	4.85	0.7	2	2.64	0.74	5	2.24	0.08	22	15.7	.07
†(I, $\lvert\Delta p\rvert$)	2	0.38	0.06	2	0.78	0.18	1	0.78	0.54	0	0.36	1.0	0	0.24	1.0	5	2.54	0.1
(I, $\lvert\Delta^2 p\rvert$)	4	0.90	0.01*	2	1.09	0.31	4	1.39	0.05	0	0.59	1.0	4	0.45	<0.01*	14	4.40	0.01*
†(I, R)	5	0.96	≤0.01*	3	0.35	0.01*	5	1.26	<0.01*	1	0.67	0.5	3	0.45	0.01*	17	3.69	<0.01*
(I, V)	3	0.97	0.07	1	1.05	0.65	3	1.87	0.29	3	1.34	0.15	4	1.1	0.02*	14	6.33	<0.01*
Sum of events for a sequence	14	8.21	<0.01*	8	3.27	0.02*	13	5.3	<0.01*	4	2.96	0.34	11	2.24	<0.01*	50	16.98	<0.01*
Sum of Binary Events Testing the Continuous in time Model																		
Sum for S Events	4	1.11	0.02	1	1.06	0.67	0	1.85	0.85	1	0.92	0.6	2	0.69	0.15	8	5.63	0.15
Sum for I Events	7	1.24	<0.01*	5	1.11	<0.01*	6	2.04	0.01*	1	1.03	0.65	3	0.69	0.03*	22	6.11	<0.01*

* Denotes significance at 5 per cent level.

† Denotes experiments testing for analytic and continuous model.

The fifth sequence (*DJI* Weekly, using *DJI* volume) was chosen to check on some rather unexpected results which appeared in the *DJI* daily data, using total market volumes.

The identifications of the events $|\Delta p|$, $|\Delta^{(2)}p|$, R were simply made with a ruler on the semi-log charts. In the case of the monthly charts (Col. 4) this accounts for the apparently unsymmetrical specification of S vs I events. For an S event, a 20% fall (4/5 p) is equal in "distance" to a 25% rise (5/4 p) on a semi-log chart.

The first three binary coincidences in Table 1 simply check what we expected from the definitions; $|\Delta p|$, $|\Delta^{(2)}p|$ and R events do not occur independently. The observed number of coincidences, N_{obs} is systematically greater than N_{theor} calculated under the assumption of independence. In most cases the excess is significant at the 5% level, denoted by (*). Summaries by sequence appear in the lower row, and by event in the last columns.

It will be noted that the significance probabilities for the summary rows and columns are computed using the fact that the sum of Poisson variables is a Poisson variable. Use of this fact requires the assumption that the individual Poisson variables summed are independent. This seems to be a reasonable assumption for the sum of events for a coincidence type (the last column) and debatable for the sum of all events for a sequence (the summary row). Obviously the numbers in one sequence of $|\Delta p|$, $|\Delta^{(2)}p|$, and R events are not independent. If and how much this error makes the significance for the sum of events for a sequence overstated (too small a value of $P(n \geq N_{\text{obs}})$, we do not know.

The second group of binary events in Table 1 check for coincidences of V with $|\Delta p|$, $|\Delta^{(2)}p|$, and R events (groups 2 and 3). An excess is expected by both the random walk model, the discrete transient model, and folklore ("volume make prices move"). In general there is a significant excess of the observed number of coincidences, N_{obs}, over N_{theor}. So we conclude that a "mix" in unknown proportions of these two models, plus folklore, is probably correct. This is certainly not a startling conclusion.

However, detailed examination of this second group of binary coincidences in Table 1 shows two rather unexpected results. Column 2 represents the daily data for the Dow Jones Industrial Average and the total market volume. These two sequences are more closely studied by the trading public than any other. For this wellspring of folklore, for *none* of the coincidences, either individually or in toto (8 observed vs 6.4 theoretical) is there a significant excess of coincidences.

We believe that this negative result can be explained in part by the following argument. The events in the Dow Jones Average were being compared with events in the total market volume sequence, and strictly speaking one should rather have used just the volume associated with the stocks in the average. This is about 7% of the total market volume.

There is in fact a certain amount of "correlation", or excess of coincidences between events in the Dow Jones Volume and total market volume, as is shown in Table 5. Nevertheless it would be preferable to compare the *DJI* with just the volume associated with the stocks in it. This comparison is made in column

TABLE 3. TRIPLE COINCIDENCES INVOLVING I AND S. COMPUTED AS COMPOUND BINARY EVENTS (EQ. 2)

Coincident Event	Sum over Sequences (1) through (4)		
	N_{obs}	N_{theor}	$P(n \geq N_{obs})$
$(S, (\|\Delta p\|, \|\Delta^{(2)}p\|))$	2	0.97	0.26
$(S, (\|\Delta p\|, R))$	1	0.98	0.63
$(S, (\|\Delta^{(2)}p\|, R))$	2	1.00	0.27
$(S, (\|\Delta p\|, V))$	0	0.73	1.0
$(S, (\|\Delta^{(2)}p\|, V))$	0	0.8	1.0
$(S, (R, V))$	0	1.14	1.0
Sum	5	5.38	0.57
Sequence	Sum over 6 triple coincidences involving S		
(1) Magnavox, daily	5	1.56	0.02*
(2) DJI Daily, Mkt., Vol.	0	.75	1.0
(3) United Airlines, weekly	0	2.29	1.0
(4) Johns Manville, monthly	0	0.80	1.0
Sum	5	5.38	0.57

Coincident Event	Sum over sequences (1)–(4)		
	N_{obs}	N_{theor}	$P(n \geq N_{obs})$
$(I, (\|\Delta p\|, \|\Delta^{(2)}p\|)$	2	1.05	0.27
$(I, (\|\Delta p\|, R))$	3	1.09	0.10
$(I, (\|\Delta^{(2)}p\|, R))$	8	1.11	<0.01*
$(I, (\|\Delta p\|, V))$	2	0.79	0.2
$(I, (\|\Delta^{(2)}p\|, V))$	3	0.65	0.03*
$(I, (R, V))$	7	1.26	<0.01*
Sum	30	11.33	<0.01*
Sequence	Sum over 6 triple coincidences involving I		
(1) Magnavox, daily	13	1.79	<0.01*
(2) DJI Daily, Market Vol.	5	0.73	<0.01*
(3) United Airlines, weekly	6	2.64	0.04*
(4) Johns Manville, monthly	1	0.97	0.63
Sum	30	11.33	<0.01*

5, using weekly data. It will be observed that in general, i.e., for the summary data for this sequence, ($N_{obs}=19$ vs. $N_{theor}=6.9$) there is a significant excess, in agreement with what was concluded from the other sequences. But in particular there is not a significant excess of $(|\Delta^{(2)}p|, V)$ coincidences ($N_{obs}=4$ vs. $N_{theor}=2.7$). This brings us to the second unexpected result in Table 1.

According to the random walk model, the volume and the sequential dispersion (measured by $|\Delta^{(2)}p|$) should increase together. This implies an excess number of ($|\Delta^{(2)}p|$, V) coincidences. The row of data in summary for this event in Table 1 ($N_{obs}=19$ vs. $N_{theor}=13.9$) does not quite significantly support this conclusion ($P=0.09$). This is a slightly embarrassing result, since in addition to the random walk model it also contradicts the predictions of the discrete transient model, and also folklore. The one exception to this embarrassment is in the first column, for an individual stock for data at daily intervals ($N_{obs}=8$ vs. $N_{theor}=3.1$).

From this limited data we can guess that folklore and the discrete transient model may be correct for daily data on individual stocks in predicting excess ($|\Delta^{(2)}p|$, V) coincidences, but not for longer basic intervals (week, month), or for an average. A better test would be to generate a synthetic random walk with variable numbers of steps (volume) between observations, and then test these data for ($|\Delta^{(2)}p|$, V) coincidences. We can guess at this stage that the excess of ($|\Delta^{(2)}p|$, V) coincidences which do occur in daily data for individual stocks are primarily due to non-random structure (folklore and the discrete transient model), and that the random walk contribution to ($|\Delta^{(2)}p|$, V) coincidences is rather small.

Let us now consider the binary coincidences involving I and S with the remaining four events (Table 2). An excess of these eight different types of coincidences (four with I and four with S) is predicted both by the discrete transient model, and also by the folklore statement, "Important moves end climatically, or on large volume".

First observe the summary figures of I and S separately. For the coincidences with S events the totals are $N_{obs}=22$ vs. $N_{theor}=15.7$, and an excess not quite significant at the 5% level ($P(N_{obs}\geq 22|N_{theor}=15.7)=0.07$. Most of the excess is provided by the individual stock at daily intervals ($N_{obs}=8$ vs. $N_{theor}=2.78$), and none of the others show a significant excess of coincidences of S with the other four events. So far as these data are concerned, we can say that the conclusion of folklore that "important moves *upward* (S events) end on large volume or climactically", applies to individual stocks and daily data (one example only), not to longer intervals, or an average.

The case for minima, or I events, is somewhat more convincing, since the difference of the totals, $N_{obs}=50$ vs. $N_{theor}=16.98$ is highly significant. The individual cases also separately show a significant excess of coincidences of I events with the remaining four, except for the monthly interval data ($N_{obs}=4$ vs. $N_{theor}=2.96$), and except for (I, $|\Delta p|$) coincidences, where the excess of coincidences is also not significant ($N_{obs}=5$ vs. $N_{theor}=2.54$).

In summary, therefore, the conclusions of folklore and the discrete transient model hold better at minima than maxima, better in individual stocks than an average, and better for intervals of a day or week, than for intervals of a month.

The asymmetry between the behavior at maxima and minima is at least in agreement with one item of folklore. Traditionally stocks tend to rally somewhat abruptly from a decline, whereas the passage through a peak is somewhat more leisurely. The above conclusions are in agreement with this statement. The same statement can be made in a variety of ways. One might say "stocks bounce something like a pingpong ball, with discontinuities at the minima of its trajectories". In market terms one can say that bargain hunting is more concentrated in time and price than profit taking. This statement is in agreement with the inference that limit orders to buy below the market are more concentrated in price than limit orders to sell above the market (ref. 6, p. 377).

There are four binary events marked with a dagger in Table 2. These are the coincidences which are expected to occur less rather than more frequently that given by independence, if a continuous in time price mechanism were correct. Examination of the data for these coincidences gives no support to such a mechanism, but merely supports our previous conclusions. Comparing N_{obs} vs. N_{theor}, there is a neglibible departure (an excess) for coincidences with S events, and a considerable excess, rather than deficiency for coincidence with the I events.

There are twelve possible triple coincidences involving I and S, which might be examined, six with I and six with S. These twelve (Table 3) are formed by combining I or S with the six binary coincidences listed in Table 1. For practically all of these twelve the calculated number of coincidences (using the form of Eq. 2) was considerably less than one. The observed number was zero for the majority of individual sequences, in the case of coincidences with S events, and only one or two in the case of coincidences with I events. Hence we give only the summary figures for these triple coincidences. Our previous conclusions from the binary coincidences are confirmed. Except for the individual stock at daily intervals ($N_{\text{obs}}=5$ vs. $N_{\text{theor}}=1.6$), there is no significant excess for S events but there is for I events, except for intervals of one month.

It should be pointed out here that N_{theor} for the triple coincidences in Table 3 are computed as though they were binary coincidences, of one single event with another single event (which happens to be binary). This method of calculation is indicated by the notation in Table 3, for example $(I(t), (R(t), |\Delta pt|))$, rather than $(I(t), R(t), |\Delta p(t)|)$ for the triple coincidence between I, R, and $|\Delta p|$ events. The reason follows from the discussion of the three groups of our six events in section 2.

It is known, or suspected, that R and $|\Delta p|$ events are not independent simply from their definitions. Thus, the expected, or theoretical number of triple coincidences as computed in Table 3, is

$$N_{\text{theor}} = \frac{N_{\text{obs}}(I)N_{\text{obs}}(R, \Delta p)\ T}{T^2}. \tag{2}$$

Specifically we do *not* use

$$N_{\text{theor}} = \frac{N_{\text{obs}}(I)N_{\text{obs}}(R)N_{\text{obs}}(|\ \Delta p\ |)T}{T^3} \tag{3}$$

which would be the expected number of triple coincidences if *all three* of the events I, R, $|\Delta p|$ were assumed under the null hypothesis to be tested, to be independent of each other.

In Table 4 are given the data for a test for delayed coincidences of volume events with I and S events. The effects are small, but the summary column at the left does give definite evidence that volume events tend to precede I and S events (*primarily* S events) and avoid following I and S events by one unit of

TABLE 4. DELAYED COINCIDENCES—BINARY V EVENTS WITH I OR S.
* DENOTES SIGNIFICANCE AT 5 PER CENT LEVEL:
$P(+) = P(n \geq N_{obs})$, $P(-) = P(n \leq N_{obs})$

Event	(1)			(2)			(3)			(4)			Sum of events for coincidence type		
	N_{obs}	N_{theor}	$P(+)$	N_{obs}	N_{theor}	$P(+)$	N_{obs}	N_{theor}	$P(+)$	N_{obs}	N_{theor}	$P(+)$	N_{obs}	N_{theor}	$P(+)$
Volume event preceding S or I event by one unit of time															
$(V(t-1), S(t)$	2	0.84	0.21	0	1.05	1.0	6	1.7	0.02*	2	1.2	0.3	10	4.79	0.02*
$(V(t-1), I(t))$	3	0.97	0.07	0	1.05	1.0	1	1.87	0.85	1	1.34	0.7	5	5.23	0.5
Sum of events for sequence	5	1.81	0.03*	0	2.0	1.0	7	3.57	0.06	3	2.54	0.5	15	10.02	0.06
Volume events following S or I event by one unit of time															
			$P(-)$			$P(-)$			$P(-)$			$P(-)$			$P(-)$
$(S(t-1), V(t))$	0	0.84	0.42	0	1.05	0.35	0	1.7	0.18	0	1.2	0.30	0	4.79	0.01*
$(I(t-1), V(t))$	0	0.97	0.36	0	1.05	0.35	2	1.87	0.7	0	1.34	0.26	2	5.23	0.09
Sum of events for sequence	0	1.81	0.15	0	2.10	0.12	2	2.57	0.51	0	2.54	0.08	2	10.02	<0.01*

TABLE 5. DATA ON COINCIDENCES BETWEEN TOTAL MARKET VOLUME AND VOLUME FOR DOW-JONES INDUSTRIAL COMPONENTS ONLY. WEEKLY DATA FROM JULY 1956 TO OCTOBER 1959, TAKEN GRAPHICALLY FROM CHARTS OF SECURITIES RESEARCH CORPORATION, BOSTON, MASS.

No. of intervals	172
No. of total market volume events V_M	33
No. of D. J. volume events V_{DJ}	23
Expected no. of $(V_M(t), V_{DJ}(t))$ coincidences $N_{\text{theor}} = \dfrac{33.23}{172} =$	4.4
Observed no. of $(V_M(t), V_{DJ}(t))$ coincidences, $N_{\text{obs}} =$ $P(n \geq N_{\text{obs}}) \ll 0.01$	14

time. The former is expressed by an excess of coincidences, the latter by a deficiency. Note that for the latter case the significance probability is for fewer than the observed number of coincidences, rather than for an excess, as in Tables 1 and 2. The slight evidence that volume events tend to precede and avoid following I and S events is an imperfect expression of the complicated and esoteric rules for volume trading signals (cf. "head and shoulders" pattern [1]).

Table 5 has already been referred to as showing a certain "correlation" (as measured by excess coincidence of volumes events) between the Dow Jones Industrial volume and the total market volume. This excess expresses a property of these two volume sequences which is quite analogous to the relation between a single stock sequence and a market average, or between two stock sequences in the same industry, say railroads or steels. In other words, these two volume sequences contain a random variable in common. The relation between correlation due to a common random variable, and excess coincidence of events due to a common random variable, would be an interesting question to explore.

DISCUSSION AND SUMMARY

The methods of this paper were devised, and the data assembled for the explicit purpose of testing for the correctness of the discrete transient method. In so doing we also tested for the random walk model. The discrete transient model was confirmed for price minima, poorly or not at all for price maxima. The volume sequence tended slightly to lead, and avoid lag in the sense of coincidences, the price sequence, but in a slightly asymmetric way. In general shorter time interval data (daily, weekly) tended to show more "non-random walk" properties than for longer intervals (monthly). The most puzzling conclusion from the data is the failure to find, except for daily data on a single stock, and excess of $(|\Delta^{(2)}p|, V)$ coincidences. This is more puzzling because an excess is predicted by the random walk model, the discrete transient model, and folklore.

In view of the imperfect agreement between theory and observation, both for random walk and discrete transient model, at the moment we can only suggest what further investigations might be undertaken to clarify these matters. Prime targets for further examination are the daily sequences on individual stocks, since our single example of this showed the most "non-random" structure.

It would also be helpful if some synthetic random walks were constructed in which the volume (number of steps per day) were varied according to either a lognormal or Pareto-Levy distribution [3]. and these synthetic sequences examined by the method of coincident events. Such a test would be helpful in showing what can be expected from such a "extended" random walk model, with variable number of steps between observations. All of this could be much better done by machine than by graphical methods, which the author finds exceedingly tedious. A computer could be readily programmed to: (a) analyze a real price and volume sequence, and (b) at the same time generate and analyze a random walk with the number of steps each day exactly equal, or proportional to the real volume sequence. The difference in the *statistical* properties of the synthetic vs. real price and volume sequences would be a good test of the non-random walk properties actually present.

APPENDIX I

The Exact and Poisson Approximation for the Occurrence of k Coincidences in T Intervals

Let us assume that n_R events R, and n_V events V occur independently of each other and the calendar date (assumptions 1–4) in a span of T days, one or none of either type of event per day. The total number of ways this may occur is

$$\frac{T!}{n_V!(T - n_V)!} \cdot \frac{T!}{n_R!(T - n_R)!} \qquad \text{A-1}$$

Out of this total number of ways, exactly k pairs, or $(R(t), V(t))$ coincidences can occur in

$$\frac{T!}{(n_R - k)!(n_V - k)!k!(T - (n_R + n_V - k))!} \qquad \text{A-2}$$

different ways. The ratio of Equation A-2 to A-1 is then the probability $P(k)$ of exactly k coincidences. Rearranged, this ratio is

$$P(k) = \left[\frac{T!}{(T - (n_R + n_V - k))!}\right]\left[\frac{n_R!}{(n_R - k)!}\right]\left[\frac{n_V}{(n_V - k)!}\right] \cdot \left[\frac{(T - n_V)!}{T!}\right]\left[\frac{(T - n_R)}{T!}\right]\frac{1}{k!} \qquad \text{A-3}$$

This is the hypergeometric distribution (Ref. 8, p. 42, with $n = T$, $n_1 = n_R$, $r = n_V$)

We repeatedly apply to the bracketed factors in A-3 the general formula

$$\frac{T!}{(T-j)!} = T^j \exp\left(\sum_{s=1}^{j-1} \log_e (1 - s/T)\right) \qquad \text{A-4}$$

Expand the logarithm and obtain, for j smaller than $\sqrt{T}$,

$$\frac{T!}{(T-j)!} \cong T^j \exp(-j^2/2T + j/2T) \qquad \text{A-5}$$

Use A-5 to express each bracket in A-3, and we ultimately find approximately

$$P(k) \cong (n_R \cdot n_V/T)^k (1/k!) \exp(-n_R n_V/T) \qquad \text{A-6}$$

an approximation good if k, n_R, n_V, are smaller than $\sqrt{T}$. Equation A-6 is just the Poisson approximation to the hypergeometric distribution we have used. (Ref. 8, p. 162)

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