considerable. The slow increase of power with mass means that large fuel loads can be carried comparatively cheaply. So mass dynamics in migrants are not determined simply by the costs of flight, but must also reflect the benefit of having sufficient fuel reserves to compensate for vagaries of the weather or other uncertainties. This makes it easier to understand how the extraordinary flight achievements of migrants might have evolved. In the case of the knot, for instance, it helps to explain how a small bird with a lean mass of only 100 grams can fly up to 5,000 km from Britain to the Russian Arctic in a single hop.

The immediate conclusion is that aerodynamic flight-power models (see refs 1 and 7, for example) do not predict total power reliably under all conditions. One possible explanation is that a bird with a full fuel load has some biomechanical means of compensating for the rise in aerodynamic power needed to support its weight, perhaps by manipulating its wing lift coefficient or by varying the rate at which it moves its wings. But it is far from obvious that any such mechanism could account for the discrepancy in power. Rather, Kvist *et al.*[3] suggest that the efficiency with which muscles convert fuel to mechanical work also rises with load for individual birds. This proposition is consistent with a calculated rise in flight efficiency with body mass between birds of different sizes[1,2], and with an increase in efficiency with flight speed in starlings[2]. But it also raises a paradox: why can't a bird carrying a small fuel load operate at the high efficiency that it can reach when carrying a large load?

The situation with fuel load may be complex, but the implications of Weimerskirch and co-workers' observations[4] on the benefits of formation flight are much clearer. Reassuringly, in this case the predictions based on aerodynamics do not fail. The idea that flying in formation gives significant aerodynamic benefit dates back to a 1914 paper by Wieselsberger[9], at the time a doctoral student under Ludwig Prandtl at Göttingen. The date is significant: the paper followed soon after Prandtl's 'lifting-line' principle of wing action of the same year, which described how vortices around the wing and in its wake are responsible for aerodynamic lift, and as such was one of the fundamental results of modern aerodynamics. The wake vortices force air downwards in the region behind the wing, but air is forced upwards outside the wake. Wieselsberger realized that this principle should apply to birds just as it did to aircraft, and that a group of birds could exploit the updraft to fly more cheaply if they adopted a V-shaped formation in which each bird flies in the up-current generated by the one in front.

Not all ornithologists have accepted the aerodynamic explanation for this spectacular flight phenomenon, however, and formation flight has been the subject of an unresolved debate between the 'aerodynamic' and 'behavioural' camps. Until now, neither side of the argument has been complete. On one hand, the aerodynamic prediction has never been fully quantified because the magnitude of the energy saving depends on the group's geometry, and the effects of flapping flight or wingbeat synchronization have not been properly modelled. On the other, there are probably benefits from travelling in a cohesive, structured group, with a dominant or experienced leading bird.

In their experiments, Weimerskirch *et al.*[4] used heart rate in great white pelicans as a proxy for energy expenditure[10]. The pelicans were trained to fly after a motor boat and light aircraft, from which measurements and observations were made. The authors found that pelicans in formation had a lower heart rate and wingbeat frequency, and glided more often. These are indications that the birds required less mechanical and total power to fly, so — qualitatively, at least — confirm the aerodynamic prediction. It is likely that aerodynamic and social benefits coevolved to establish this common flight behaviour in large birds.

Taken together, these two sets of measurements[3,4] add considerably to what we can say about the energetics of bird flight. Aerodynamic models are, in general, quite good at predicting the biomechanical aspects of flight, but they cannot yet be extended reliably to predict total flight power. We know far too little of the internal physiological processes in flight, and in particular about the efficiency with which the flight muscles generate mechanical work. It is not yet possible to predict this efficiency reliably, or to explain how it varies with mass or speed. If we are to understand the energetics of flight in any bird under any conditions, or are to be able to use simple aerodynamic models to predict total flight power, we will first have to know a lot more about how flight muscle works. ■

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Mathematics

Where drunkards hang out

Ian Stewart

The trail of a particle undergoing brownian motion might be unkindly described as a drunken walk. A 40-year-old conjecture related to brownian motion and such random walks has finally been proved.

The drunkard's walk, more soberly known as a random walk, has long been a mainstay of probability theory. The drunkard starts from a lamppost and takes random steps forwards or backwards. Then where does he go? If the probability is the same for both directions, he will return infinitely often to the lamppost, but on average will take infinitely long to get there. Mathematicians have generalized this situation to a random walk on a regular lattice in two, three or even more dimensions. A random walk happens on a discrete lattice, but an analogous process occurs in continuous space. This is brownian motion, in which the random jiggling movements of particles in a fluid suspension are attributed to collisions with fluid molecules. A conjecture[1] related to both of these situations — first posed by Paul Erdös and S. James Taylor in 1960 — has now been proved[2] in a paper in *Acta Mathematica* by Amir Dembo, Yuval Peres, Jay Rosen and Ofer Zeitouni.

For random walks in higher dimensions there are some important differences. For example, the drunkard returns to the lamppost with 100% probability for walks in one and two dimensions, but with a lower probability for three dimensions or higher. If you are lost in a desert and wander at random, eventually you'll get back to where you started. But if you're 'Lost In Space', you may not. The theory of random walks goes back to the late nineteenth century and the pioneers of combinatorial probability theory. But the specific problem under discussion originated more than forty years ago, when Erdös and Taylor posed a question about random walks on a square lattice in the plane. How many times does the walker revisit the most frequently visited site in the first *n* steps? In other words, how many times does the drunkard go to his favourite watering hole?

Erdös and Taylor were able to make significant progress towards an answer. They

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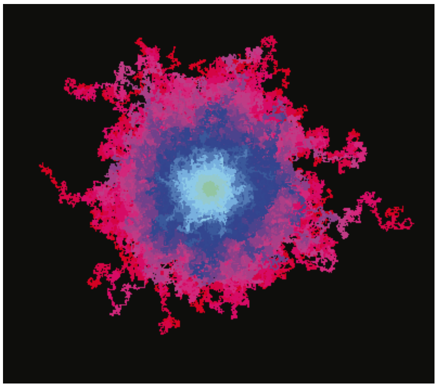

Figure 1 **Random walks and fractals. A contour map of the territory covered by 500 random walkers, who started together at the centre. The different colours of the pixels reflect how often a pixel is visited. The roughness of the contour surfaces appears to be the same however many times they are visited. This 'self-similarity' is one of the defining features of fractals — the idea that if we shrink or enlarge a fractal pattern its appearance should remain unchanged. This random walk is isotropic, so its distribution has circular symmetry. The path of a particle undergoing brownian motion in the plane is analogous to a random walk, and a 40-year-old conjecture related to brownian motion and random walks has just been proved by Dembo *et al.*[2].**

proved that for a large number of steps, n, the number of visits to the most frequently visited site lies between $(\log n)^2/4\pi$ and $(\log n)^2/\pi$. They conjectured that the larger of these numbers is the correct answer. (Curious that π should turn up here, but more on that later.) The methods used by Dembo *et al.*[2] to prove this conjecture are taken from fractal geometry: the core of the proof is a study of the fine multifractal structure of brownian motion. So their results constitute an important and rigorous application of fractals to probability theory and mathematical physics. Brownian motion is no longer important in its original physical context, but it has many other applications, including some to the financial sector.

In 1987, E. A. Perkins and Taylor obtained upper and lower bounds analogous to those of Erdös and Taylor for brownian motion in the plane. Given a most-frequently-visited disc of fluid, they tried to find the fraction of time during which the randomly jiggling particle is inside that disc. They proved that the answer lies between a certain expression and the same expression divided by four, and conjectured that the larger of these provides the correct answer. Their results were clear analogues of those of Erdös and Taylor. Dembo *et al.*[2] also prove the Perkins–Taylor conjecture, and again show that the upper bound is the correct answer. Indeed, they work mainly on brownian motion, and then transfer their results to random walks.

These results tell us a great deal about the statistical properties of these processes. For example, mathematics can describe the probability distribution of fluctuations (departures from the average behaviour), which obey power-law statistics, and the value of the limit determined by Dembo *et al.* appears in the power laws governing their behaviour. The authors also reveal some finer detail. For example, they prove that the most frequently visited points consistently lie near the boundary of the region that the random walk visits. It seems that the drunkard's favourite hangout is as far away from the lamppost as he can get.

Last year, the same authors studied brownian motion in three dimensions, where the key insight turned out to be a localization effect. Spheres of fluid that are most frequently visited gain most of their visits during a relatively short interval of time. Essentially, the drunkard gets inside the sphere and stays there for a while. This does not happen in two dimensions — instead, the drunkard makes frequent excursions away from the most frequently occupied disc, but keeps returning to it. These excursions occur on all length scales, which is where fractal geometry comes in. The main technical issue is how the lengths of these excursions away from the disc, on a given scale, relate to the fraction of time for which the disc is occupied. This relationship is multifractal — it can be represented by a family of fractals whose fractal dimension (a measure of their roughness) is not constant[3].

Where does that π come from? As a fundamental constant in mathematics, π turns up all over the place, from geometry to number theory. Often the connection to its original definition involving circles is obscure. But in this case there is a clue in the repeated reference to occupation of discs and spheres. Brownian motion is isotropic: all directions are treated equally. So the probability distributions associated with brownian motion have circular symmetry. The same is approximately true for random walks (Fig. 1) when the number of steps, n, is large. Once circular discs enter the analysis, π cannot be far away. But this is not something the drunkard would expect to stumble upon. ■

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Archaeology

Tree trail to Chaco Canyon

Jared Diamond

Strontium isotopes have been used to identify the sources of timber in buildings around one thousand years old. The method can now help to solve a range of other problems.

The aim of 'provenance studies' is to identify the geographical source of a material by measuring some chemical or physical property (a 'signature') that is known to vary geographically. As they describe in *Proceedings of the National Academy of Sciences*, English *et al.*[1] have developed a method for finding the origin of plant materials by means of their strontium isotope ratio — a signature already widely used as an environmental tracer by geologists, hydrologists, ecologists and archaeologists[2–5]. English *et al.* apply the method to an archaeological mystery: the origin of the big roof timbers at the famous ancient Native American site of Chaco Canyon (Fig. 1, overleaf), which stands in a now-treeless desert of the southwestern United States[6]. The results demonstrate not only the existence of a complex regional economic system in one part of the ancient world, but also a new method that can be applied more broadly.

Archaeologists had already worked out other provenance methods to identify trade networks and migration routes. In particular, geographical variation in the trace-element composition and isotopic ratios of rocks and clays had been used to locate sources of stone tools and pottery. For instance, proto-Polynesian stone tools from a site on Fiji were found to be made of obsidian from the island of New Britain 4,500 km to the west, proving the existence of a long-distance trade network 3,000 years ago[7]. As for biological materials, archaeologists have attempted to identify the sources of human[8] and animal teeth, but very little work has been done on plant materials.

The main challenge in these studies is to select a signature that varies on a geographical scale appropriate to the problem of interest. For example, discriminating between potential sources 100 km apart requires a signature that varies in the environment on that scale, rather than on a scale of 1,000 km (yielding no difference between the potential sources) or of 1 km