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RANDOM WALK AND FORWARD EXCHANGE RATES: A SPECTRAL ANALYSIS

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This paper examines the random-walk hypothesis in the forward exchange market by applying spectral analysis to the three-month forward rates for dollars against sterling in the period 1961-1967.

I. Random Walk and Anticipatory Prices

Holbrook Working and Paul Cootner show that successive changes in an anticipatory price series can be expected to be independent, and in this situation, the price series itself describes a random walk. Alternatively, if successive price changes are dependent, then the market has elements of inefficiency that could result from a corner on the market or an uneven availability of information across market participants. This random walk hypothesis has been examined extensively in securities markets. Tests have been run between the null hypothesis of randomness and many formulations of alternative (serial) dependency hypotheses, including those of cycles, trends, and composites such as Cootner's specification of barriers. From these tests, researchers conclude that there are few statistically significant departures from randomness. In the cases where departures do occur, the profitability of the strategies which would take advantage of the nonrandomness is not high enough to attract resources.

II. Spectral Analysis

Assuming certain assumptions are met, spectral analysis of time series can provide tests of hypotheses relating to dependency. Because comprehensive treatment of spectral analysis of economic time series is available in the works of Granger and Hatanaka, as well as Jenkins and Watts, this section is deliberately brief.

Events in a time series may be independent of one another or dependent on the next adjacent event or on the second, third . . .nth adjacent event. The relationships between events may be examined by computing the autocovariance function (or power spectrum) of the time series and analyzing it into the

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frequencies (or more loosely, the cycles) of which it is composed. Thus in a weekly-recorded time series, there may be 2, 3, ...n weekly cycles generating the observations. Spectral analysis requires that the time series meet, or be transformed to meet, the condition of stationarity. Given this condition, spectral analysis can lead to the identification, if they exist, of frequency-bands which contribute significantly more than proportionately to the total power spectrum.

III. Null Hypothesis

A forward exchange rate is a price, determined now, at which a future transaction will be concluded. Such a price is an anticipatory or speculative price, and if the requirements of an efficient market are met, then successive changes in forward exchange rates should be independent, or, in other words, forward exchange rates may be expected to perform a random walk. This independence or random-walk concept is the null hypothesis in this paper.

IV. Alternate Hypothesis

There are many possible sources of nonrandomness in a forward exchange market. From time to time, central banks collaborate with the intention of changing the price from what it would be otherwise.[1] Such activity may be carried on secretly (and only revealed *ex post*), in which case the information requirements of an efficient market are not met. It may also be executed with ultimate access to resources substantially larger than those of other participants, which is akin to the operation of a corner in a commodity market. The fact that central banks are active in these ways may impact on the general availability of forward contracts so that seasonal or other regular movements in forward rates are imperfectly smoothed out by speculatory and hedging activity.[2] Central banks also may proscribe certain forward exchange market activity, a rule-making power not available to other market participants. Theories of the determination of forward exchange rates relate such rates to the spot rate and to expected yields on equal risk investments in the two countries. Any nonrandomness in these time series may be reflected in the forward exchange rate data. This is not an

[1]Such collaboration is described in the six-monthly reports of Treasury and Federal Reserve foreign exchange operations, and in the quarterly reports of the Bank of England.

[2]Central banks' price-manipulating activities could also be a source of shorter-term nonrandomness. Working [11, p. 160] states, ". . . if the futures prices (market expectations) are subject only to necessary inaccuracy, the price changes will be completely unpredictable. Changes which are completely unpredictable are, by definition, random." When central banks collaborate to alter a forward exchange rate, they predict future price changes and have the predictions fulfilled.

exhaustive catalog of possible sources of nonrandomness; it suffices to indicate that there are plausible, although at present unproven, sources. The existence of sources such as these would be indicated by dependence among changes in forward exchange rates. This formulation of the alternate hypothesis of nonrandomness does not specify the length of lag in the possible dependency. However, the use of spectral analysis permits testing of the random-walk hypothesis, and if significant nonrandomness is found, the specification of important periodicities.

V. Data

The forward exchange rates analyzed are those for U.S. dollars in London, i.e., the dollar/pound midpoint rate, for dollars three months forward, as reported in *International Financial Statistics*. The data in this paper comprise the rate each Friday; when Friday is a public holiday, the rate for the nearest trading day is used. Thus a set of 359 weekly observations -- commencing Friday, December 30, 1960, and ending Friday, November 17, 1967 -- is compiled. This covers the period from explicit adoption of external convertibility (I.M.F. Article VIII Status) through the Friday preceding devaluation of the pound from \$2.80 to \$2.40 on November 18, 1967.

First differences between successive observations of weekly forward exchange rates are computed, and the resultant time series is the input for the analysis. The mean of the distribution of first differences is -0.0065 cents (i.e., minus six-thousandths of a cent; see Figure I). The distribution is comprised of many observations ($n = 358$) and is approximately symmetrical, and inspection of the data indicates that this time series meets the requirement of approximate stationarity.[3]

VI. Analysis

Figure II presents a graph of a spectral density function in which all frequencies contribute equally to the total power spectrum. This graph corresponds to the null hypothesis specified above. When empirical data are used, the spectral density estimated for each frequency would not be expected to be absolutely equal, at 2.0, as in Figure I.[4] Instead, at a 95 percent confidence level, at least 95 percent of the frequencies would be expected to have spectral density estimates whose confidence bounds include 2.0.

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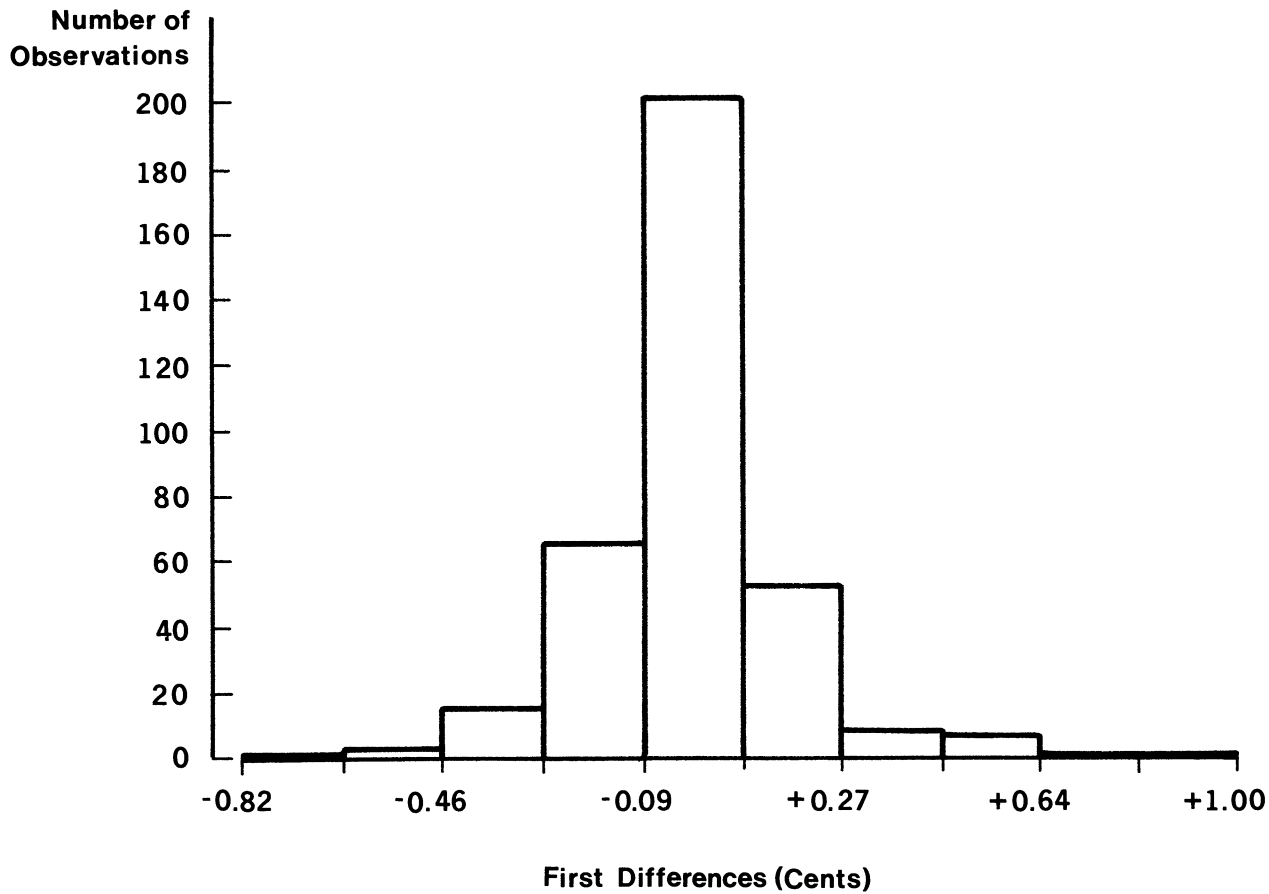
[3] *A priori*, the variance is likely to be stationary, since the related spot rate is limited to movements within plus or minus 1 percent of par.

[4] 2.0 because the highest frequency is 0.5 (2 cycles per week), and the spectral density function integrates to unity.

FIGURE I

FREQUENCY DISTRIBUTION OF FORWARD EXCHANGE RATE FIRST DIFFERENCES

MEAN - 0.0065 CENTS, STD. DEV. 0.195 CENTS, SKEWNESS (β_1) 0.101, KURTOSIS (β_2) 7.66



Source: Based on International Monetary Fund, *International Financial Statistics*, 1961-1967.

FIGURE II

GRAPH OF SPECTRAL DENSITY FUNCTION

ILLUSTRATING NULL HYPOTHESIS

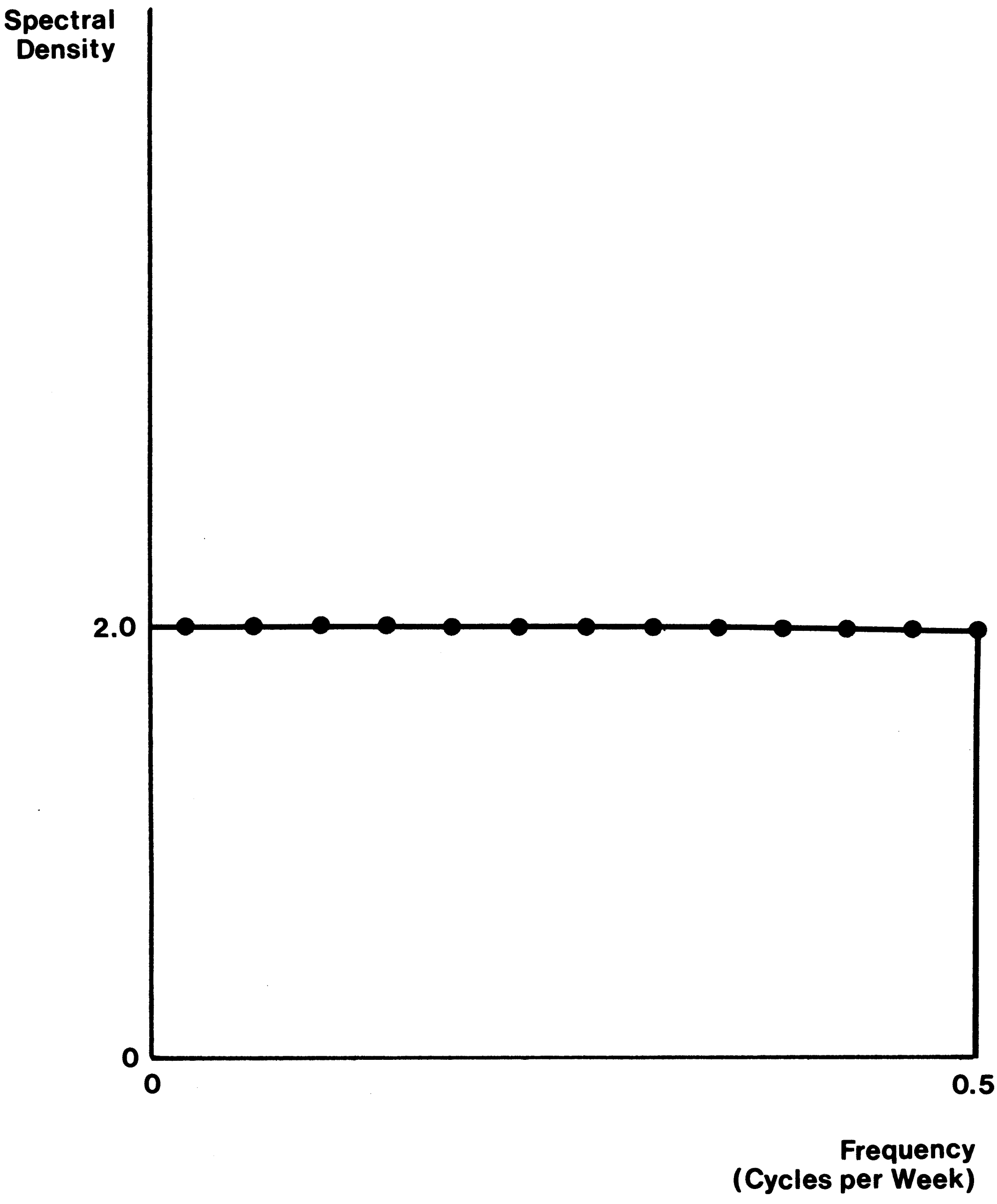


Figure III presents a graph of the spectral density function of the forward exchange rate data, together with a graphic presentation of a test of the hypotheses specified earlier. The vertical bars show (each at a 97.5 percent confidence level) the lower width of the confidence bound for frequencies with high (> 2.0) spectral density estimates, and the upper width of the confidence bound for frequencies with low (< 2.0) spectral density estimates. Of the 31 frequencies, 0.0156 through 0.48 cycles per week,[5] eight have either upper or lower confidence bounds which do not include 2.0. This proportion, 8/31 or 25.8 percent, is greater than the expected proportion of 5 percent attributable to chance. Thus a departure from randomness is concluded, significant at a 5 percent level.

Figure I also shows three peaks, at frequencies of 0.03125, 0.26563, and 0.40625 cycles per week. These peaks indicate the existence of cycles of length 32, 3.8, and 2.5 weeks. Such cycles would not exist if changes in forward exchange rates were randomly distributed. Some possible causes of these cycles are stated above in Section IV.

VII. The Possibility of Infinite Variance[6]

The distribution of first differences shown in Figure I, above, is very leptokurtic ($B_2 = 7.66$) compared to a normal distribution. Mandelbrot and Fama show that some other speculative prices also have significantly leptokurtic distributions and may be more appropriately modeled by an infinite variance model. Accordingly, it is necessary to investigate the possibility of infinite variance, because spectral analysis is applicable only to situations in which variance is finite (or can be assumed to be finite). If the nonrandomness observed above is present when the whole series is analyzed but disappears when extremely high and low values (the tails of the distribution in Figure I) are excluded, then finite variance is a doubtful assumption.

As a check on the finite variance assumption, the analysis is repeated after truncating the original series of first differences. The truncation removes the largest and smallest 5 percent of the data (18 observations in total) and removes much of the leptokurtosis (B_2 is now 3.73). The truncated series yields similar results to the original series, using exactly the same procedure as before for computing the spectral density. Of the 31 frequencies, 4 have either upper or lower confidence bounds which do not include 2.0. This proportion, 4/31 or 12.9 percent, differs from that attributable to chance. Thus although the spectral density function of the truncated series is somewhat flatter than that of the original series, a departure from randomness at a 5

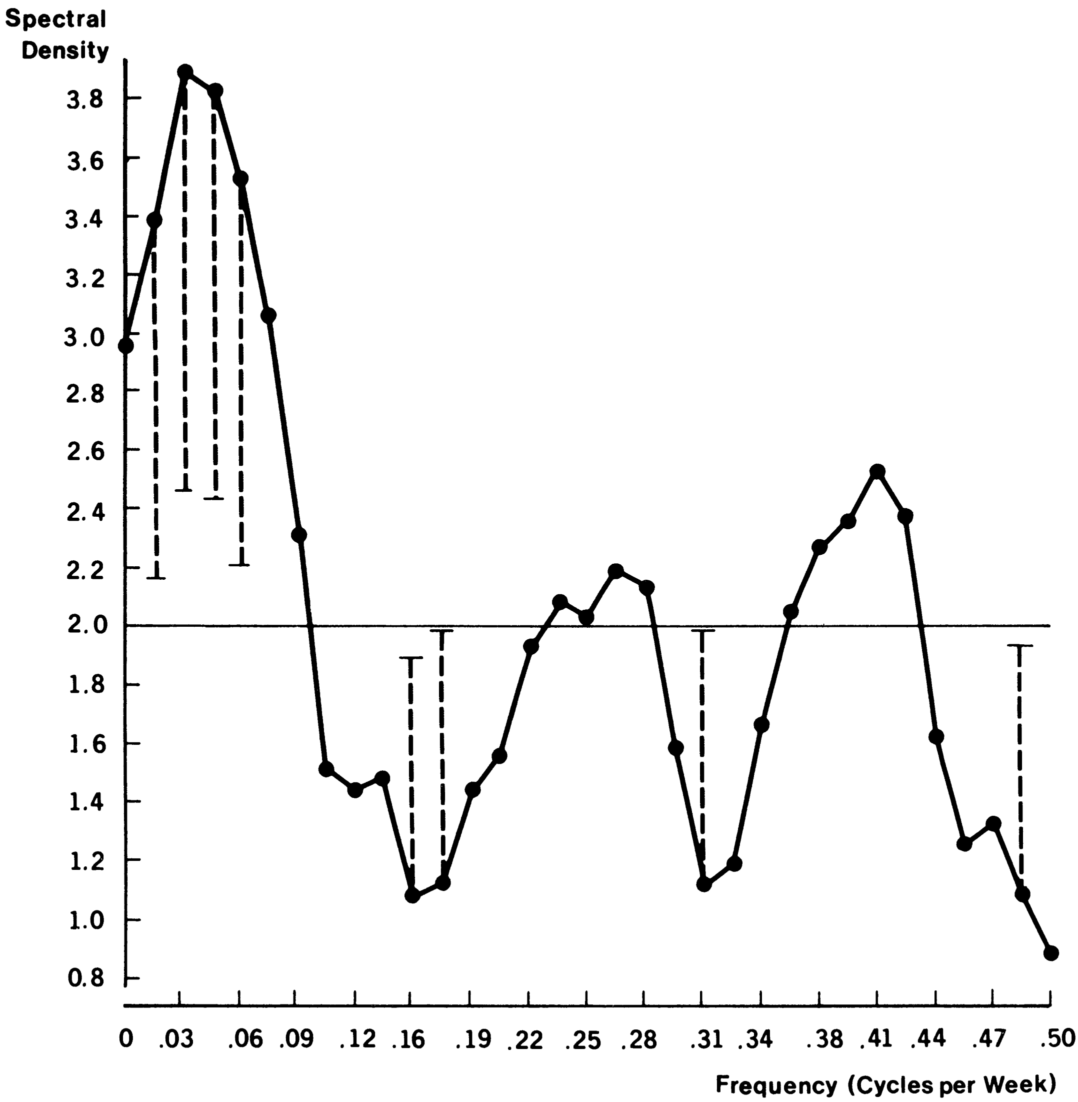
[5]Estimates for frequencies 0 and 0.5 excluded. A frequency of 0 has no meaning, and the estimate for 0.5 has fewer degrees of freedom than the estimates for the other frequencies.

[6]The author thanks the anonymous referee for suggesting the inclusion of this section.

FIGURE III

GRAPH OF SPECTRAL DENSITY FUNCTION, FORWARD EXCHANGE RATE DATA.

SPECTRAL ESTIMATES ARE COMPUTED WITH A MAXIMUM LAG OF 32 WEEKS FOR A SERIES OF 358 DIFFERENCES, USING A TUKEY-HANNING WINDOW



percent level is still observed. Consequently, a finite variance model appears appropriate for this series of first differences in forward exchange rates.[7]

VIII. Conclusions and Implications

The preceding analysis shows that pound-dollar 90-day forward exchange rates between 1961 and 1967 do not behave in a manner consistent with the random-walk hypothesis. There are two major implications of a significant departure from randomness. First, there are some inefficiencies in the forward exchange market analyzed. Second, periodicity is present in the time series.

Knowledge of the periodicities provided in this paper (representing cycles of 32, 3.8, and 2.5 weeks) may assist others in the development of exchange rate forecasts and the development of successful trading rules. In this context it is interesting to note that, in Herbert Grubel's study of foreign exchange speculation, the highest return trading rule employs a three-week lag.[8] This lag is similar to the short-run periodicities observed in the present analysis.

Finally, this paper examines the appropriateness of analyzing these forward exchange rate data with a finite variance model. Both the original and the truncated series exhibit significant nonrandomness, implying that the process generating the series does not behave as though it has infinite variance.

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[7]This conclusion is similar to that of Godfrey, Granger, and Morgenstern [5, p. 13].

[8]For this rule Grubel [7, p. 106] assumes that "spot rates often move in clearly discernible cycles with definite upper and lower bounds (the intervention points), resembling sine waves with rates of change slower the greater the proximity to the intervention points." The operation of the rule is based on forward rates and the three preceding weekly average spot rates. Grubel tested the rule on data for 1955 through 1961.

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